Ph.D. Thesis

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Carmichael's Conjecture and the Unit Group Function

Abstract

The Euler φ-function and Carmichael λ-function are extremely important in modern number theory, and much work has been devoted to studying the distribution and arithmetic properties of the values of each function. One interesting unresolved question is Carmichael's Conjecture which states that there is no value of φ with a unique preimage. Recently, the analogous question for λ has been studied with some success, but neither is completely settled.

As $\varphi(n)$ and $\lambda(n)$ are both determined by the structure of the unit group modulo n, it is of interest to study the distribution of unit groups. Among other results, we give an upper bound for the number of non-isomorphic unit groups with order at most x, and we give a lower bound for the maximum number of distinct integers n having isomorphic unit groups for $\varphi(n) \leq x$. We also consider the distribution of ordered pairs $(\varphi(n), \lambda(n))$ where $\varphi(n) \leq x$. For this function and the unit group function, we address the analogue of Carmichael's Conjecture by showing that there are infinitely many counterexamples.