Refocusing of Null-Geodesics in Lorentz Manifolds

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Abstract: We investigate weak and strong refocusing of light rays in a space-time and related concepts. A strongly causal space-time \((X^{n+1}, g)\) is strongly refocusing at \(x \in X\) if there is a point \(y \neq x\) such that all null-geodesics through \(y\) pass through \(x\). A space-time is strongly refocusing if it is strongly refocusing at some point.

Robert Low introduced three definitions of (weak) refocusing. We prove that these definitions are indeed equivalent. Following a sketch provided by Low, we give a thorough proof of his statement that a strongly causal non-refocusing space-time is homeomorphic to its sky space.

A strongly refocusing space-time is refocusing. The converse is unknown. We construct examples of space-times which are refocusing, but not strongly so, at a particular point. These space-times are strongly refocusing at other points. The geometrization conjecture proved by Perelman implies that a globally hyperbolic refocusing space-time of dimension \(\leq 4\) admits a strongly refocusing Lorentz metric.

We show that the set of points at which a strongly causal space-time is refocusing is closed. We prove that a Lorentz covering space of a strongly causal refocusing space-time is a strongly causal refocusing space-time. This generalizes the result of Chernov and Rudyak for globally hyperbolic space-times.

We compare refocusing and strong refocusing with their Riemannian analogues, \(\check{Y}^x\)- and \(\check{Y}_t^x\)-manifolds. A complete connected Riemannian manifold \(M\) is called a \(\check{Y}_t^x\)-manifold if there exist \(x \in M\) and \(l \in \mathbb{R}^+\) such that all unit speed geodesics starting at \(x\) at time 0 return to \(x\) at time \(l\). In our work with Chernov and Sadykov we introduce \(\check{Y}^x\)-manifolds that generalize \(Y_t^x\)-manifolds. There we prove that some conclusions of the Bérard-Bergery Theorem for \(Y_t^x\)-manifolds hold in fact for \(\check{Y}^x\)-manifolds. This result is discussed in this thesis.

Following the sketch of Chernov we provide the thorough proof of the statement in their paper with Rudyak that a timelike curve in a globally hyperbolic space-time can be perturbed so that it is transverse to a null-cone and avoids the singular and multiple points of the null-cone. We investigate a possible generalization.