The Bloch spectrum of a quantum graph Ralf Rueckriemen

Abstract

We show that families of leafless quantum graphs that are isospectral for the standard Laplacian are finite. We show that the minimum edge length is a spectral invariant. We give an upper bound for the size of isospectral families in terms of the total edge length of the quantum graphs.

We define the Bloch spectrum of a quantum graph to be the map that assigns to each element in the deRham cohomology the spectrum of an associated magnetic Schrödinger operator. We show that the Bloch spectrum determines the Albanese torus, the block structure and the planarity of the graph. It determines a geometric dual of a planar graph. This enables us to show that the Bloch spectrum identifies and completely determines planar 3-connected quantum graphs.