

# $K$ -theory for Twisted Groupoid $C^*$ -Algebras

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*Abstract*

This thesis constitutes the first steps in the author's program to investigate the question of when a homotopy of 2-cocycles  $\omega = \{\omega_t\}_{t \in [0,1]}$  on a locally compact Hausdorff groupoid  $\mathcal{G}$  induces an isomorphism of the  $K$ -theory groups of the reduced twisted groupoid  $C^*$ -algebras:

$$K_*(C_r^*(\mathcal{G}, \omega_0)) \cong K_*(C_r^*(\mathcal{G}, \omega_1)).$$

Generalizing work of Echterhoff, Lück, Phillips, and Walters, I show that if  $\mathcal{G} = G \ltimes X$  is a transformation group, then whenever  $G$  satisfies the Baum-Connes conjecture with coefficients and  $X$  is compact, a homotopy  $\omega = \{\omega_t\}_{t \in [0,1]}$  of 2-cocycles on  $G \ltimes X$  gives rise to an isomorphism

$$K_*(C_r^*(G \ltimes X, \omega_0)) \cong K_*(C_r^*(G \ltimes X, \omega_1)).$$

In addition, I build on work by Kumjian, Pask, and Sims to show that if  $\mathcal{G} = \mathcal{G}_\Lambda$  is the infinite path groupoid associated to a row-finite higher-rank graph  $\Lambda$  with no sources, and  $\{c_t\}_{t \in [0,1]}$  is a homotopy of 2-cocycles on  $\Lambda$ , then

$$K_*(C_r^*(\mathcal{G}_\Lambda, \sigma_{c_0})) \cong K_*(C_r^*(\mathcal{G}_\Lambda, \sigma_{c_1})),$$

where  $\sigma_{c_t}$  denotes the 2-cocycle on  $\mathcal{G}_\Lambda$  associated to the 2-cocycle  $c_t$  on  $\Lambda$ .