

BIJECTIVE COMBINATORICS OF REDUCED DECOMPOSITIONS

ABSTRACT. We study the bijective combinatorics of reduced words. These are fundamental objects in the study of Coxeter groups. We restrict our focus to reduced words of permutations and signed permutations.

Our results can all be situated within the context of two parallels. The first parallel is between the enumerative theory of reduced words and that of Coxeter group elements. The second parallel is between the combinatorics of permutations and that of signed permutations.

The enumerative properties of reduced words have been studied widely since Stanley first proposed the problem of counting reduced words of permutations. One approach is via the Little map, a bijection introduced by David Little. We introduce a Little map for reduced words of signed permutations and explain how to think of these maps as an analogue jeu de taquin for the setting of reduced words.

We also extend dual equivalence and Rothe diagrams graphs to the setting of signed permutations. In the former case, this leads to a new description of reduced words for signed permutations as balanced labelings of diagrams.

Much of the work introduced in this thesis appeared for the first time in papers joint with Benjamin Young, and with Sara Billey, Austin Roberts and Benjamin Young.