Generalized Fourier Transforms and their Applications

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Abstract

This thesis centers around a generalization of the classical discrete Fourier transform. We first present a general diagrammatic approach to the construction of efficient algorithms for computing the Fourier transform of a function on a finite group or semisimple algebra. By extending work which connects Bratteli diagrams to the construction of Fast Fourier Transform algorithms [2], we make explicit use of the path algebra connection to the construction of Gel'fand-Tsetlin bases and work in the setting of general semisimple algebras and quivers. We relate this framework to the construction of a *configuration space* derived from a Bratteli diagram. In this setting the complexity of an algorithm for computing a Fourier transform reduces to the calculation of the dimension of the associated configuration space.

We give explicit counting results to find the dimension of these configuration spaces, and thus the complexity of the associated Fourier transform. Our methods give improved upper bounds for the general linear groups over finite fields, the classical Weyl groups, and homogeneous spaces of finite groups, while also recovering the best known algorithms for the symmetric group and compact Lie groups. We extend these results further to semisimple algebras, giving the first results for non-trivial upper bounds for computing Fourier transforms on the Brauer and Birman-Murakami-Wenzl (BMW) algebras.

The extension of our algorithm to Fourier transforms on semisimple algebras is motivated by emerging applications of such transforms. In particular, Fourier transforms on the Iwahori-Hecke algebras have been used to analyze Metropolis-based systematic scanning strategies for generating Coxeter group elements [1]. We consider the Metropolis algorithm in the context of the Brauer and BMW monoids and provide systematic scanning strategies for generating monoid elements. As the BMW monoid consists of tangle diagrams, these scanning strategies can be rephrased as random walks on links and tangles. We translate these walks into left multiplication operators in the corresponding BMW algebra. Taking this algebraic perspective enables the use of tools from representation theory to analyze the walks; in particular, we develop a norm arising from a trace function on the BMW algebra to analyze the time to stationarity of the walks.

References

[1] P. Diaconis and A. Ram, *Analysis of systematic scan Metropolis algorithms using Iwahori-Hecke algebra techniques*, Michigan Math. J. **48** (2000), 157–190.

[2] D. Maslen and D. Rockmore, *Separation of variables and the computation of Fourier transforms on finite groups. I*, J. Amer. Math. Soc. **10** (1997), no. 1, 169–214.