Boundary integral methods and their applications

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Abstract

Boundary integral methods have long been used to solve boundary value problems for elliptic partial differential equations with piecewise constant coefficients, since they have several numerical advantages over conventional volume discretization. They reduce the dimensionality of the problem by one and usually produce schemes compatible with fast algorithms, like the Fast Multipole Method. In this thesis, we use the boundary integral approach to study three problems. For the first problem, we solve the Laplace boundary value problem in a doubly periodic planar domain and as a result obtain the effective conductivity of periodic composite materials. Our use of proxy points for periodizing is much simpler than existing approaches using periodic Green's functions. For the second problem, we extend the scheme to when the governing differential equation is the Stokes' system describing viscous flow. In the last problem, we compute the Laplacian spectrum for smooth planar domains by searching for roots of the Fredholm determinants of boundary integral operators. Our method is robust even when the domain is not simply-connected and requires less computation per eigenvalue in the spectrum than standard methods in the literature. All of our methods have spectral convergence rate on smooth domains and greatly enhanced efficiency with the use of fast algorithms specific to each scheme, and thus enable large-scale computation for complex geometries.