Matched Products and Dynamical Models for Multiplex Networks

Daryl R. DeFord

Abstract

This thesis analyzes models for multiplex networks, which are generalizations of graphs that allow for multiple types of connections between the objects of interest. This is a rapidly expanding field that has recently become a prominent topic in network science. Much of this interest has centered around models that use a tensorial formulation to describe multiplex data and generalize tools developed for single layer networks.

We begin by discussing some of the implications of these tensorial approaches and formulating a graph–theoretic product that represents the structure of the unfolded tensor operators. In addition to exploring the network properties of these models, we also connect them to common combinatorial questions and apply them to generate families of graphs whose Stirling numbers of the first kind can be computed recursively.

The main contribution of the thesis is to introduce a new family of operators, motivated by dynamical processes, for describing the properties of multiplex networks. This dynamical framework is very flexible and allows us to generalize many commonly studied models for network dynamics to the setting of multiplex data. Our analysis of these dynamics includes spectral bounds derived in terms of the component layers. Additionally, we show that in several settings this model interpolates the behavior of two extreme models, one where the individual layers do not interact and one where the layers are aggregated into a single, simple network.

After motivating and developing this dynamical approach we use it to explore centrality and clustering on multiplex network data. These measures have deep connections to dynamics and we are able to exploit these relationships in order to apply them in the multiplex setting. Although modularity has become a common approach for community detection in temporal multiplex networks, this thesis also develops methods for performing spectral clustering in the general multiplex setting. We conclude with applications to two real–world data sets drawn from sociology and economics. These applications demonstrate the flexibility and power of our model, uncovering behavior that could not be observed in more traditional single layer analysis.