# Permutations: descents, cycles, and patterns

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*Abstract*

This doctoral thesis is drawn from the author's work as a graduate student, investigating the number of permutations with certain properties.

One main topic is the enumeration of cyclic permutations according to descent set. Using a result of Gessel and Reutenauer, we find a simple formula for the number of cyclic permutations with a given descent set, by expressing it in terms of ordinary descent numbers (i.e., those counting all permutations with a given descent set). We then use this formula to show that, for almost all sets $I \subseteq [n-1]$, the fraction of size-$n$ permutations with descent set $I$ which are $n$-cycles is asymptotically $1/n$. As a special case, we recover a result of Stanley for alternating cycles. We also use our formula to count $n$-cycles with no two consecutive descents.

The other main topic is the enumeration of the centrosymmetric permutations in a pattern-avoiding permutation class. A permutation is centrosymmetric if it is fixed by a half-turn rotation of its diagram. We investigate the question of whether the growth rate of a permutation class equals the growth rate of its even-size centrosymmetric elements. We present various examples where the latter growth rate is strictly less, but we conjecture that the reverse inequality cannot occur. We conjecture that equality holds if the class is sum closed, and we prove this conjecture in the special case where the growth rate is at most $\xi \approx 2.30522$.