2-group Belyi Maps

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Abstract

This thesis concerns the explicit computation of Galois Belyi maps $\phi\colon X \to \mathbb{P}^1$ with monodromy group a 2-group, which we call 2-group Belyi maps. The computation has two parts. The first is a combinatorial computation to enumerate the isomorphism classes of 2-group Belyi maps. The second part is an explicit algorithm to compute equations for the algebraic curve X and the Belyi map ϕ.

The motivation behind computing these maps comes from Beckmann's theorem, which relates the primes of bad reduction of X to the primes dividing the order of the monodromy group of ϕ. Beckmann's theorem also implies that the field of moduli of a 2-group Belyi map is unramified away from 2. Are these moduli fields always solvable? Is the field generated by the 2-power torsion subgroup of the Jacobian of X solvable over $\mathbb{Q}$? This work aims to provide the computational framework to begin answering these questions.