Penney's game for permutations

Yixin Lin

Abstract

We explore the permutation analog of Penney's game for words. Two players, in order, each choose a permutation of length $k \geq 3$. Then, a sequence of independent random values from a continuous distribution is generated until the relative order of the last k numbers matches one of the chosen permutations, declaring the player who selected that permutation as the winner. We calculate the winning probabilities for all pairs of permutations of length 3 and some pairs of length 4. We also provide formulas for computing the winning probabilities more generally and conjecture a winning strategy for the second player when k is arbitrary.

We also consider a Markov chain variation of Penney's game for permutations. After two players have selected their permutations of length k, the game starts at any permutation of length k with probability $1/k!$. At each step, we transition from the current permutation to the next one with probability $1/k$, provided that the relative order of the last $k-1$ numbers of the current permutation coincide with the first $k-1$ numbers of the next permutation. We provide a formula for computing the expected time to observe any permutation for the first time (known as the hitting time) and discuss some conditions under which two permutations have the same hitting time. Furthermore, we compute the winning probabilities of all pairs of permutations of any length and conjecture a non-losing strategy for the second player.