The average order of elements in the multiplicative group of a finite field

Yilan Hu
Advisor: Carl Pomerance
Dartmouth College
Hanover, NH 03755 USA
yilan.hu@dartmouth.edu

March 14, 2010

Abstract

For any positive integer, we investigate the average order of elements in the multiplicative group of a finite field $\mathbb{F}_{p^k}$, where p is prime. Luca worked out the case $k = 1$ and we extend his results to the case $k = 2$ and sketch the outline to approach the general case where k is arbitrary but fixed. We show that the mean of the average order function for $p \leqslant x$ is asymptotically $C_k x^k$ for a certain positive constant C_k.