

Unitary untouchable numbers

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Abstract

In 1973, Erdős proved that a positive proportion of numbers are untouchable; that is, not of the form $s(n)$, where $s(n) := \sigma(n) - n$ is the sum of the proper divisors of n . We investigate the analogous question where σ is replaced with similar divisor functions, such as the unitary sum-of-divisors function σ^* (which sums those divisors d of n co-prime to n/d). We use the slightly modified version of Erdős's original argument from the aforementioned work to prove that there are infinitely many unitary untouchable numbers. In one of the cases, the theory of covering congruences makes a surprising appearance.