Math 11 Fall 2007
Some Practice Questions for Exam II

The usual warnings apply: This is not a "practice exam." It is not intended to reflect the length or balance of an actual exam, and not all the material we covered necessarily is tested here.

1. Short answer questions.

   (a) TRUE or FALSE?

$$\int_a^b \int_c^d \frac{\partial f}{\partial x}(x, y)\, dy\, dx = f(b, d) - f(a, c)$$

   (b) Give the best answer:

$$\int\int_R f(x, y)\, dA$$

   is guaranteed to exist when R is a closed rectangle in the xy-plane ($a \leq x \leq b$ and $c \leq y \leq d$) and the function f

      i. is defined at every point of R.
      ii. is continuous at every point of R.
      iii. is differentiable at every point of R.
      iv. None of the above conditions will guarantee the integral exists.

   (c) Rewrite the integral with the variables in the opposite order.

$$\int_{-1}^1 \int_{x^2}^1 f(x, y)\, dy\, dx$$

   (d) Rewrite this polar coordinate integral using rectangular coordinates:

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{1}{\cos\theta}} r^2\, dr\, d\theta$$

   (e) Find and classify all critical points of the function

$$f(x, y) = x^2 + 8y^2 + 4xy - 4x.$$

(f) TRUE or FALSE?

$$\int_0^1 \int_0^1 \sin(x^2)\sin(y^2)\,dy\,dx = \left[\int_0^1 \sin(x^2)\,dx\right]^2$$

2. Express

$$\iiint_E x\,dV,$$

where E is the region above the xy-plane and below the downward-facing cone $z = 1 - \sqrt{x^2+y^2}$, as an iterated integral in

(a) rectangular

(b) cylindrical

(c) spherical

coordinates. You do not need to evaluate the integral.

3. Find the maximum and minimum values of the function

$$f(x,y) = x^2 + 2y^2 + 3x$$

on the region $x^2 + y^2 \leq 4$.

4. Find the point(s) at which the graph of the function

$$f(x,y) = e^{-x^2-2y^2}$$

is steepest (that is, the point(s) at which the slope of the graph, in the direction of maximal slope, is as large as possible.)

5. Find the volume of the region inside the sphere of radius 2 centered at the origin and above the plane $z = 1$.

6. Write down a double integral (or a sum of double integrals) representing the volume of the portion of the first octant above the plane $z = 2x+2y$ and below the surface $z = 3 - x^2 - y^2$. Do not evaluate the integral.

7. A spherical solid of radius 1 centered at the origin has mass density at point P given by

$$1 + (\text{distance from } P \text{ to } z\text{-axis})^2.$$

Find its total mass.