

# Hour Exam #2

## Math 3

Nov. 9, 2009

Name: \_\_\_\_\_

Instructor (circle):

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**Instructions:** You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form, and on page 1 of your exam booklet. You may write on the exam, but you will only receive credit on Scantron (multiple choice) problems for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 12 multiple choice problems each worth 6 points and 2 long-answer written problems worth 14 points each. Check to see that you have 8 pages of questions plus the cover page for a total of 9 pages.

Non-multiple choice questions:

Problem	Points	Score
13	14	
14	14	
Total	28	

1. For the function  $f(x) = \ln(e^x + 1)$ , we have  $f'(x) =$

(a)  $\frac{1}{e^x + 1}$

(b) 1

(c)  $1 + e^{-x}$

(d)  $\frac{e^x}{e^x + 1}$

(e) none of the above

2. An anti-derivative of a function  $f(x)$  is a function  $g(x)$  for which  $g'(x) = f(x)$ . Which function below is an anti-derivative for  $\ln(x)$ ?

(a)  $x \ln(x)$

(b)  $\frac{1}{2} \ln 2(x)$

(c)  $x \ln(x) - x$

(d)  $\frac{1}{x}$

(e) none of the above

3. Consider the tangent line to the graph of  $y = 2^x$  at  $(1, 2)$ . The slope is

- (a)  $> 1$
- (b)  $= 1$
- (c) in the interval  $(0, 1)$
- (d)  $\leq 0$
- (e) the slope doesn't exist

4. Consider the curve  $x^3 + xy + 2y^3 = 4$  and the point  $(1, 1)$  on the curve. The equation of the tangent line to the curve at  $(1, 1)$  is

- (a)  $y = -\frac{4}{7}x + \frac{11}{7}$
- (b)  $y = -x + 2$
- (c)  $y = \frac{1}{3}x + \frac{2}{3}$
- (d) there is no tangent line at this point
- (e) none of the above

5. Use the linearization technique to give an approximation to  $\sqrt{101}$ . One gets

- (a) 10.01
- (b) 10.05
- (c) 10.1
- (d) 10.5
- (e) this problem is not amenable to the linearization technique

6. Say you are solving the equation  $x^3 + x = 1$  with Newton's method, and your initial trial solution,  $x_0$ , is 0. What is  $x_2$ ?

- (a)  $\frac{2}{3}$
- (b)  $\frac{3}{4}$
- (c)  $\frac{59}{86}$
- (d) 1
- (e) none of the above

7. Find  $\int x^2 + x + 1 dx$ . It is

- (a)  $2x + 1 + C$
- (b)  $\frac{1}{2}x^3 + \frac{1}{3}x^2 + x + C$
- (c)  $\frac{1}{3}x^3 + \frac{1}{2}x^2 + \ln(x) + C$
- (d)  $\frac{1}{4}x^3 + \frac{1}{3}x^2 + x + C$
- (e) none of the above

8. Solve the IVP:  $\frac{dy}{dx} = x^2y^2, y(1) = 1$ .

- (a)  $y = x^3$
- (b)  $y = \frac{1}{x^3}$
- (c)  $y = e^{x^3-1}$
- (d)  $y = \frac{3}{4 - x^3}$
- (e) none of the above

9. Give the general solution to the differential equation  $\frac{dy}{dx} = y + 1$ .

(a)  $y = Ce^x - 1$

(b)  $y = e^x + C$

(c)  $y = \frac{1}{2}x^2 + x + C$

(d)  $y = e^x + x + C$

(e) none of the above

10. A radioactive substance decays so that the rate of decay is proportional to the amount present. After 4 years, exactly one-third of the radioactive substance remains. The half-life of the substance is

(a) less than 2 years

(b) exactly 2 years

(c) more than 2 years, but less than 4 years

(d) more than 4 years

(e) cannot be determined from the given information

**11.** For the function  $f(x) = x^3 - 3x + 1$ , one of following holds; which one? [Note: There is a gray area on whether finite endpoints should be included in an interval for this type of problem; but this is a very minor thing. So, for example, if you like answer (a), but you'd prefer to see the intervals written as  $(-\infty, 0)$  and  $(0, \infty)$ , then choose (a). And similarly for the other choices.]

- (a) it is decreasing on  $(-\infty, 0]$  and increasing on  $[0, \infty)$
- (b) it is increasing on  $(-\infty, 0]$  and decreasing on  $[0, \infty)$
- (c) it is concave up on  $(-\infty, 0]$  and concave down on  $[0, \infty)$
- (d) it is concave down on  $(-\infty, 0]$  and concave up on  $[0, \infty)$
- (e) it is increasing on  $(-\infty, \infty)$

**12.** A 10' ladder is leaning against a wall but sliding down, with the bottom of the ladder moving from the wall at  $t + 1$  feet per second at time  $t$ . At time  $t = 2$  seconds, the base is 8' from the wall. How fast is the top of the ladder falling at this instant?

- (a) 3' per second
- (b) 4' per second
- (c) 5' per second
- (d) 6' per second
- (e)  $5\frac{1}{3}$ ' per second

**13.** Use Euler's method with step size 1 to approximate  $y(3)$  given that  $y(-1) = 0$  and  $\frac{dy}{dx} = x + 2y$ .

14. Sketch the graph of  $y = x^3 + 3x^2 - 2$ , indicating in a side statement the intervals where the function is increasing, where it is decreasing, where it is concave up, and where it is concave down.