Hour Exam #1

Math 3

Oct. 20, 2010

Name:____

On this, the first of the two Math 3 hour-long exams in Fall 2010, and on the second hour-exam, and on the final examination I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:____

Instructor (circle):

Lahr (Sec. 1, 8:45)   Franklin (Sec. 2, 11:15)   Diesel (Sec. 3, 12:30)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on page 1 of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 15 multiple-choice problems worth 4 points each and 3 long-answer written problems worth a total of 40 points. Check to see that you have 11 pages of questions plus the cover page for a total of 12 pages.

Non-multiple choice questions:

Problem	Points	Score
16	10	
17	15	
18	15	
Total	40	

1. The domain of the function $f(x) = \frac{x^2+x-12}{x+4}$ is

(a) $(-\infty, -4) \cup (-4, \infty)$. (correct)

(b) $(-\infty, \infty)$.

(c) $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$.

(d) $(-\infty, 4) \cup (4, \infty)$.

(e) none of the above.

2. For the same function f as in Problem 1, the horizontal and vertical asymptotes are

(a) $y = 0$ and $x = 0$.

(b) $y = 0$ and $x = -4$.

(c) $x = 3$ and $y = -4$.

(d) $x = -4$.

(e) none of the above. (correct)

3. A weight is suspended by a spring above the floor. Its height in meters above the floor at time t seconds is given by the formula

$$h(t) = 4 + \frac{2}{\pi}\cos(\pi t).$$

Note that the function $h(t)$ has the derivative

$$h'(t) = -2\sin(\pi t).$$

What is the weight doing at time $t = 1$ second?

(a) moving upward

(b) moving downward

(c) accelerating downward

(d) accelerating upward (correct)

(e) none of the above

4. The function $f(x) = 3^x + 3^{-x} + 3$ is

(a) even. (correct)

(b) odd.

(c) neither even nor odd.

(d) both even and odd.

5. Let $f(x) = \ln(x^2)$ and $g(x) = e^{x+1}$. Find $g(f(x))$.

(a) e^{x-1}

(b) $e \cdot x^2$ (correct)

(c) $2(x+1)$

(d) $x^2 + 1$

(e) none of the above

6. For the composite function in Problem 5, find the domain of $g(f(x))$.

(a) $(-\infty, 0) \cup (0, \infty)$ (correct)

(b) $(-\infty, \infty)$

(c) $(0, \infty)$

(d) $(-\infty, 1) \cup (1, \infty)$

(e) none of the above

7. Consider the rational function $f(x) = \frac{2x^3+4x+1}{5x^a - x^3+6}$, where a is a constant. When is the following statement true?

$$\lim_{x\to\infty} f(x) = \infty$$

(a) always

(b) sometimes

(c) never (correct)

8. Let $f(x) = \frac{x^2+2x-8}{x^2-16}$. Let A be $\lim_{x\to 4} f(x)$ and B be $\lim_{x\to -4} f(x)$. Which of the following is true?

(a) $A = \frac{1}{4}$ and $B = \frac{3}{4}$.

(b) A and B do not exist.

(c) $A = \frac{1}{4}$ and B does not exist.

(d) A does not exist and $B = \frac{3}{4}$. (correct)

(e) none of the above

9. Let $f(x) = \frac{x}{7+x}$. Find the inverse function $f^{-1}(x)$.

(a) $\frac{7+x}{x}$

(b) $\frac{y}{7+y}$

(c) $\frac{7x}{1-x}$ (correct)

(d) $\frac{7x}{x-1}$

(e) none of the above

10. Find the domain and range of the function $f(x)$ from Problem 9.

(a) domain $= (-\infty, -7) \cup (-7, \infty)$, range $= (-\infty, \infty)$

(b) domain $= (-\infty, -7) \cup (-7, \infty)$, range $= (-\infty, 0) \cup (0, \infty)$

(c) domain $= (-\infty, \infty)$, range $= (-\infty, -7) \cup (-7, \infty)$

(d) domain $= (-\infty, -1) \cup (-1, \infty)$, range $= (-\infty, 1) \cup (1, \infty)$

(e) none of the above (correct)

11. Find $f'(x)$ if $f(x) = 12x^2 + \frac{\pi^3}{3}$.

(a) $24x + \pi^2$

(b) $24x$ (correct)

(c) $24 + \frac{\pi^3}{3}$

(d) 24

(e) none of the above

12. Consider the function

$$f(x) = \begin{cases} x^2 & x < -1 \\ 4 & x = -1 \\ x + 2 & x > -1 \end{cases}$$

Which of the following statements is true of f?

(a) $f(x)$ is continuous everywhere on its domain.

(b) $f(x)$ is continuous at $x = -1$, but $\lim_{x \to -1} f(x) \neq f(-1)$.

(c) $f(x)$ is discontinuous at $x = -1$, and this discontinuity is not removable.

(d) $f(x)$ is discontinuous at $x = -1$, and this discontinuity is removable. (correct)

(e) none of the above

13. Find the derivative of $f(x) = x^2 \sin(4x)$.

(a) $2x\cos(4x)$

(b) $2x\sin(4x) + x^2\cos(4x)$

(c) $2x\sin(4x) + 4x^2\cos(4x)$ (correct)

(d) $2x\sin(4x) - x^2\cos(4x)$

(e) none of the above

14. Find $f'(x)$ when $f(x) = \frac{\tan(6x)}{x^2+3x}$.

(a) $\frac{6(x^2+3x)\sec^2(6x)-(2x+3)\tan(6x)}{(2x+3)^2}$

(b) $\frac{6(x^2+3x)\sec^2(6x)-(2x+3)\tan(6x)}{(x^2+3x)^2}$ (correct)

(c) $\frac{6\sec^2(6x)}{2x+3}$

(d) $\frac{6(2x+3)\sec^2(6x)}{(x^2+3x)^2}$

(e) none of the above

15. Let

$$f(x) = \begin{cases} -2x & x \le 0 \\ 2x & x > 0 \end{cases}$$

Does the derivative of f exist at $x = 0$? Choose the correct answer *and the best reason for it.*

(a) Yes, because $\lim_{x\to 0} f(x)$ exists and equals $f(0)$.

(b) Yes, because $\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h}$ and $\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h}$ both exist.

(c) No, because f is not continuous at $x = 0$.

(d) No, because $\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} \neq \lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h}$. (correct)

END OF MULTIPLE-CHOICE PROBLEMS

PROBLEMS 16, 17, AND 18 ARE LONG-ANSWER PROBLEMS.

16. The function $f(x) = (x^2+1)^{-\frac{1}{3}}$ has the derivative $-\frac{2x}{3}(x^2+1)^{-\frac{4}{3}}$. Find an equation for the tangent line to the graph of $y = f(x)$ at the point $(\sqrt{7}, \frac{1}{2})$. Show all of your work and explain your steps.

Solution: The slope of the tangent line to the graph of f at $x = a$ is the value $f'(a)$, so the slope of the tangent line to the graph of f at $x = \sqrt{7}$ is

$$f'(\sqrt{7}) = -\frac{2\sqrt{7}}{3}\left(\left(\sqrt{7}\right)^2 + 1\right)^{-\frac{4}{3}} = -\frac{2\sqrt{7}}{3}(8)^{-\frac{4}{3}} = -\frac{\sqrt{7}}{24}.$$

Now we use the point-slope formula to find an equation for the line with slope $-\frac{\sqrt{7}}{24}$ passing through the point $(\sqrt{7}, \frac{1}{2})$:

$$y - \frac{1}{2} = -\frac{\sqrt{7}}{24}\left(x - \sqrt{7}\right)$$

17. Calculate the derivative of $f(x) = 3x^2 - 5$ using the formula

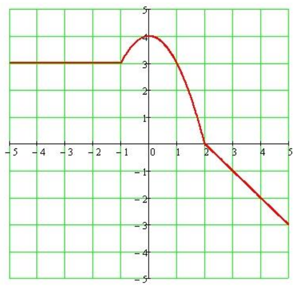
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

You must use the limit definition of the derivative and show all your work to receive any credit for this problem.

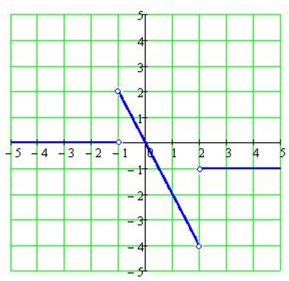
Solution:

$$\begin{aligned}
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{(3(x+h)^2 - 5) - (3x^2 - 5)}{h} \\
&= \lim_{h \to 0} \frac{(3x^2 + 6xh + 3h^2 - 5) - (3x^2 - 5)}{h} \\
&= \lim_{h \to 0} \frac{6xh + 3h^2}{h} \\
&= \lim_{h \to 0} (6x + 3h) \\
&= 6x
\end{aligned}$$

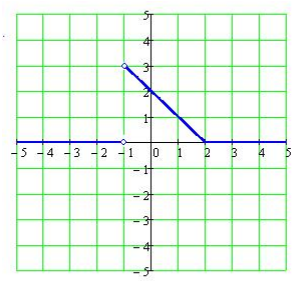
18. Here is a graph of a function.



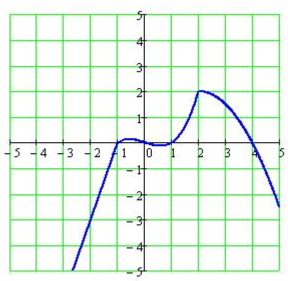
Which graph below could be the graph of its derivative? Circle the letter of the most likely graph and explain your choice in a few sentences.



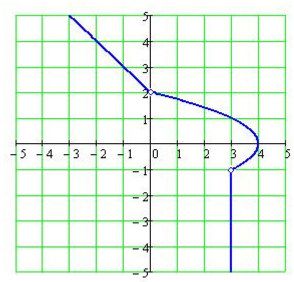
(a)



(b)



(c)



(d)

Solution: Graph (a) is the most likely graph of the derivative of the function. The slope of the graph of the original function is 0 for $x < -1$, positive for $-1 < x < 0$, 0 at $x = 0$, negative for $0 < x < 2$, and -1 for $x > 2$. Since the derivative represents the slope of the graph, we want to find the one graph of the four with these properties, and graph (a) is the only one that has all of them.