

# Hour Exam #2

## Math 3

Nov. 8, 2010

Name: \_\_\_\_\_

On this, the second of the two Math 3 hour-long exams in Fall 2010, and on the final examination I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature: \_\_\_\_\_

Instructor (circle):

Lahr (Sec. 1, 8:45)    Franklin (Sec. 2, 11:15)    Diesel (Sec. 3, 12:30)

**Instructions:** You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on page 1 of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 11 multiple-choice problems worth 6 points each and 3 long-answer written problems worth a total of 34 points. Check to see that you have 9 pages of questions plus the cover page for a total of 10 pages.

Long-answer questions:

Problem	Points	Score
12	10	
13	10	
14	14	
Total	34	

1. Let  $f(x) = e^{\sin(x)}$ . What is  $f'(x)$ ?

(a)  $\ln(\sin(x)) \cdot \cos(x)$

(b)  $e^{\cos(x)}$

(c)  $\cos(x) \cdot e^{\sin(x)}$  (correct)

(d)  $\sin(x) \cdot e^{\sin(x)-1}$

(e) none of the above

2. Which function below is an antiderivative for  $\frac{x}{\sqrt{3x^2-4}}$ ?

(a)  $\frac{x^2}{\sqrt{x^3-4x+C}}$

(b)  $\frac{\sqrt{3x^2-4}}{6}$  (correct)

(c)  $6x\sqrt{3x^2-4}$

(d)  $\frac{x^2\sqrt{3x^2-4}}{6}$

(e) none of the above

3. A ball is thrown into the air from a height of 4 feet. Its height in feet above the ground  $t$  seconds after being thrown is

$$h(t) = -16t^2 + 64t + 4,$$

and the derivative of  $h$  is

$$h'(t) = -32t + 64.$$

What is the maximum height reached by the ball?

- (a) 16 feet
- (b) 64 feet
- (c) 68 feet (correct)
- (d) 196 feet
- (e) none of the above

4. Find the equation of the line tangent to the curve  $y = \log_2(x + 1)$  at the point  $(1, 1)$ .

- (a)  $y = \frac{x}{2\ln 2} - \frac{1}{2\ln 2} + 1$  (correct)
- (b)  $y = \frac{x}{2} + \frac{1}{2}$
- (c)  $y = \frac{\ln 2}{2}x - \frac{\ln 2}{2} + 1$
- (d) There is no tangent line at this point.
- (e) none of the above

5. Find the linearization of  $y = \sqrt[3]{x}$  at the point  $(27, 3)$ .

(a)  $y = \frac{1}{27}(x - 27) + 3$  (correct)

(b)  $y = \frac{1}{9}(x - 27) + 3$

(c)  $y = \frac{1}{3}(x - 3) + 27$

(d)  $y = \frac{1}{27}(x - 27) - 3$

(e) none of the above

6. A root of  $f(x) = \sin(x) - \frac{x}{2}$  is to be solved using Newton's method. If your initial guess is  $x_0 = \pi$ , what is the value of  $x_2$ ?

(a) 1

(b)  $-\frac{2\pi}{3} + 2\sqrt{3}$

(c)  $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$  (correct)

(d)  $\frac{\pi}{3}$

(e) none of the above

7. Find  $\int(x^3 + 2x - 1)dx$ .

(a)  $3x^2 + 2 + C$

(b)  $\frac{1}{4}x^4 + x^2 - x + C$  (correct)

(c)  $\frac{1}{3}x^4 + \frac{1}{2}x^2 - x + C$

(d)  $\frac{1}{4}x^4 + \frac{1}{2}x^2 - 1 + C$

(e) none of the above

8. Solve the initial value problem  $\frac{dy}{dx} = x^2y$ ;  $y(0) = 4$ .

(a)  $y = -\frac{1}{x} + 4$

(b)  $y = \frac{x^3}{3} + 4$

(c)  $y = e^{\frac{x^3}{3} + \ln 4}$  (correct)

(d)  $y = e^{\frac{x^3}{3} + 4}$

(e) none of the above

9. Give the general solution of the differential equation  $\frac{dy}{dx} = 2y$ .

- (a)  $2x + C$
- (b)  $x^2 + C$
- (c)  $Ce^{2x}$  (correct)
- (d)  $e^{2x} + C$
- (e) none of the above

10. A population of Snigglers increases at a rate proportional to the size of the population. Two years after a team of biologists begins observing the Snigglers, there are three times as many as there were when the biologists started their observations. What is the doubling time of the Sniggler population?

- (a)  $e^{\frac{2}{\sqrt{3}}}$  years
- (b) 1 year
- (c)  $\frac{2}{\sqrt{3}}$  years
- (d)  $\frac{\ln 2}{\ln \sqrt{3}}$  years (correct)
- (e) none of the above

11. Which of the following is true for the function  $f(x) = x^3 - 12x + 17$ ?

(a)  $f$  is increasing and concave up on  $[2, \infty)$ . (correct)

(b)  $f$  is increasing and concave down on  $[2, \infty)$ .

(c)  $f$  is decreasing and concave up on  $[2, \infty)$ .

(d)  $f$  is decreasing and concave down on  $[2, \infty)$ .

(e) none of the above

END OF MULTIPLE-CHOICE PROBLEMS

PROBLEMS 12, 13, AND 14 ARE LONG-ANSWER PROBLEMS.

12. A stone is thrown into a pond, and waves start moving out from it in a circle whose radius is increasing at a rate of  $2 \frac{m}{s}$ . How fast is the area of the circle formed by the waves increasing when the radius of the circle is  $5 m$ ?

Let's call the area of the circle created by the waves  $A$  and the radius of this circle  $r$ . We know that  $\frac{dr}{dt} = 2 \frac{m}{s}$ , and we are interested in this circle at the time when the radius is  $5 m$ . We can relate the area and the radius using the equation  $A = \pi r^2$ . When we implicitly differentiate both sides of this equation with respect to time, we get

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

Now we can solve for  $\frac{dA}{dt}$  at the right moment:

$$\frac{dA}{dt} = 2\pi(5 m) \left(2 \frac{m}{s}\right) = 20\pi \frac{m^2}{s}$$



13. Use Euler's method with step size 1 to approximate  $y(4)$  given that  $\frac{dy}{dx} = xy$  and  $y(0) = 2$ .

We have  $x_0 = 0$  and  $y_0 = 2$ .

$$\begin{aligned}x_1 &= x_0 + \Delta x = 0 + 1 = 1 \\y_1 &= y_0 + (x_0 y_0) \Delta x = 2 + (0 \cdot 2)(1) = 2\end{aligned}$$

$$\begin{aligned}x_2 &= x_1 + \Delta x = 1 + 1 = 2 \\y_2 &= y_1 + (x_1 y_1) \Delta x = 2 + (1 \cdot 2)(1) = 4\end{aligned}$$

$$\begin{aligned}x_3 &= x_2 + \Delta x = 2 + 1 = 3 \\y_3 &= y_2 + (x_2 y_2) \Delta x = 4 + (2 \cdot 4)(1) = 12\end{aligned}$$

$$\begin{aligned}x_4 &= x_3 + \Delta x = 3 + 1 = 4 \\y_4 &= y_3 + (x_3 y_3) \Delta x = 12 + (3 \cdot 12)(1) = 48\end{aligned}$$

This means that our estimate for  $y(4)$  is 48.

14. Sketch the graph of  $y = \frac{1}{x^2+3}$ . Indicate in a side statement the intervals where the function is (a) increasing, (b) decreasing, (c) concave up, and (d) concave down. Please use the grid provided.

