Hour Exam #2

Math 3

November 7, 2011

Name (Print): Solutions

Last First

On this the second of two Math 3 hour exams in Fall 2011, and on the final examination I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:____

Instructor (circle):

Lahr (Sec. 1, 8:45) Crytser (Sec. 2, 11:15) Daugherty (Sec. 3, 12:30)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on the cover of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 12 multiple choice problems worth 5 points each and 3 long-answer written problems worth a total of 40 points. Check to see that you have 10 pages of questions plus the cover page and a blank page at the end for scrap work.

Non-multiple choice questions:

Problem	Points	Score
13	10	
14	10	
15	20	
Total	40	

1. If $f(x) = x^{\sin(x)}$, what is $f'(x)$?

   (a) $f'(x) = \sin(x)x^{\sin(x)-1}$.

   (b) $f'(x) = \cos(x)x^{\sin(x)}$

   (c) $f'(x) = \sin(x) + x\cos(x)$

   (d) $f'(x) = (\cos(x)\ln(x) + \sin(x)\frac{1}{x})x^{\sin(x)}$

   (e) none of the above

let $y = x^{\sin(x)}$

so $\ln(y) = \sin(x)\ln(x)$

$\frac{d}{dx}$

$\frac{1}{y}y' = \cos(x)\ln(x) + \frac{1}{x}\sin(x)$

so $y' = x^{\sin(x)}(\sim)$

2. If $f(x) = \ln(x^2 + 3)$, what is $f'(x)$?

   (a) $f'(x) = \frac{2x}{x^2+3}$

   (b) $f'(x) = \frac{1}{x^2+3} + C$

   (c) $f'(x) = \frac{-1}{(x^2+3)^2}$

   (d) $f'(x) = e^{x^2+3}$

   (e) none of the above

$\frac{1}{x^2+3} \cdot \frac{d}{dx}(x^2+3)$

3. If $f(x) = 3^x + \log_3(x)$, what is $f'(x)$?

(a) $f'(x) = 3^x + \frac{1}{3x}$
(b) $f'(x) = 3^x + \frac{1}{\ln(3)x}$
(c) $f'(x) = x3^{x-1} + \frac{1}{3x}$
(d) $f'(x) = \ln(3)3^x + \frac{1}{\ln(3)x}$
(e) none of the above

$a^x = e^{\ln(a)x}$ $\qquad \frac{d}{dx}(a^x) = \ln(a)\, a^x$

$\frac{\ln(x)}{\ln(3)} = \log_3(x)$ $\qquad \frac{d}{dx}(\log_3(x)) = \frac{1}{\ln(3)\,x}$

4. Calculate $\int x^3 - 5x^2 + x^{-1} dx$. $= \frac{x^4}{4} - 5\frac{x^3}{3} + \ln|x| + c$

(a) $3x^2 - 10x - x^{-2} + C$
(b) $\frac{x^4}{4} - \frac{5x^3}{3} + |x| + C$
(c) $\frac{x^4}{4} - \frac{5x^3}{3} - x^{-2} + C$
(d) $\frac{x^4}{4} - \frac{10x}{3} + \ln(|x|) + C$
(e) none of the above

5. If a ball is shot upwards at a speed of 5 m/s, how high is it after 3 seconds? (Recall the acceleration due to gravity is $g = 9.8\text{m/s}^2$.)

(a) $5 + 9.8(3)$ meters

(b) $\frac{9.8}{2}3^2 + (5)(3)$ meters

(c) $-\frac{9.8}{2}5^2 + (5)(3)$ meters

(d) $-\frac{9.8}{2}3^2 + 5$ meters

(e) $-\frac{9.8}{2}3^2 + (5)(3)$ meters

$y = -\frac{9.8}{2}t^2 + 5t + 0$

$f'(x) = 2x$

6. Use Newton's method to approximate a root of $f(x) = x^2 - 7$, using $x_0 = 3$ by calculating x_2.

(a) $\frac{8}{3} - \frac{(8/3)^2-7}{2(8/3)}$

(b) $3 - \frac{2}{6}$

(c) $\frac{3}{2-\frac{1}{3}}$

(d) $\frac{8}{3} + \frac{(8/3)^2-7}{2(8/3)}$

(e) $\frac{3^2-7}{2(3)}$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - (3^2-7)/6$

$= 3 - 1/3 = \boxed{8/3}$

$x_2 = \underset{8/3}{\underset{\downarrow}{x_1}} - \frac{(8/3)^2-7}{2(8/3)}$

7. Does the mean value theorem guarantee that there is some point on the interval $(1, 2)$ where the derivative of $f(x) = \frac{x+1}{x^2}$ is equal to $\frac{-5}{4}$.

   (a) Yes, because f is continuous on $(1, 2)$
   (b) Yes, because f is differentiable on $(1, 2)$
   (c) Yes, because f is continuous on $[1, 2]$ and differentiable on $(1, 2)$
   (d) No, because f has a removable discontinuity at $x = 0$
   (e) No, because f has does not have a tangent line at $x = 3/2$

8. If the length of a cube is changing at a rate of 5 in/s, how fast is the volume changing when the length of the side of the cube is 2in?

   (a) 125 cubic inches per second
   (b) 60 cubic inches per second
   (c) 25 cubic inches per second
   (d) 12 cubic inches per second
   (e) 8 cubic inches per second

$V = \ell^3$

$\downarrow d/dt$

$\frac{dV}{dt} = 3\ell^2 \cdot \frac{d\ell}{dt} = 3 \cdot 2^2 \cdot 5$

$= 6 \cdot 10$

9. An object heats or cools at a rate proportional to the the difference between the temperature of the object and the ambient temperature. Suppose a potato, which has been cooled to 35 degrees Fahrenheit, is pulled out of a refrigerator and put into a 72-degree warm room. Let y(t) be the temperature of the potato at time t. Which of the following best models the temperature y starting from the time it's pulled out of the refrigerator?

(a) $\frac{dy}{dt} = ky - 35$, $y(0) = 72$

(b) $\frac{dy}{dt} = y - 72$, $y(0) = 35$

(c) $\frac{dy}{dt} = ky - 72$, $y(0) = 35$

(d) $\frac{dy}{dt} = k(y - 35)$, $y(0) = 72$

(e) $\frac{dy}{dt} = k(y - 72)$, $y(0) = 35$

10. Which is a solution to the differential equation $\frac{d^2y}{dx^2} = y$?

(a) $y = e^{2x}$

(b) $y = e^{-x}$

(c) $y = \cos(x)$

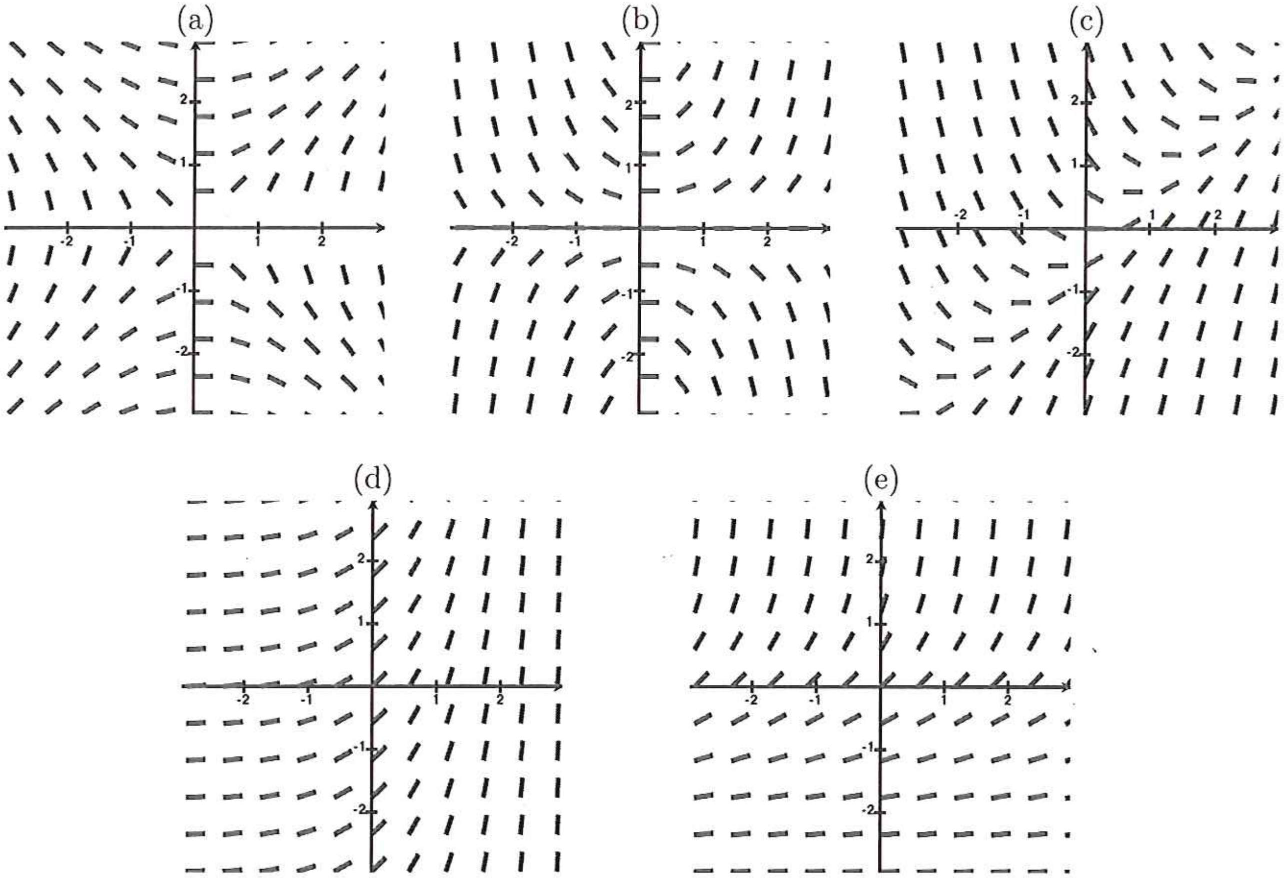
(d) $y = \sin(x)$

(e) none of the above

$\frac{d^2}{dt^2} e^{at} = a^2 e^{at}$

So $\frac{d^2}{dt^2} e^{-t} = (-1)^2 e^{-t}$

For the next two problems, refer to the following five slope fields.



11. Which of the above is the slope field for the differential equation $\frac{dy}{dx} = xy$? b

0 on x & y axes

12. Which of the above is the slope field for the differential equation $\frac{dy}{dx} = e^x$? d

increases as x increases

Long answer questions

13. Solve

$$\frac{dy}{dx} = \cos(3x)y.$$

separate:

$$\int \frac{1}{y}\,dy = \int \cos(3x)\,dx$$

$$\ln|y| = \qquad = \frac{1}{3}\sin(3x) + c$$

so $\quad |y| = e^{\frac{1}{3}\sin(3x)+c}$

$$= e^{c} e^{\frac{1}{3}\sin(3x)}$$

so $\quad \boxed{y = A e^{1/3 \sin(3x)}}$

where A is a constant.

Check: $\frac{d}{dx} \underbrace{A e^{\frac{1}{3}\sin(3x)}}_{y} = A\left(\frac{1}{3}\cos(3x) * 3\right) e^{\frac{1}{3}\sin(3x)}$

$$= \cos(3x) * \underbrace{A e^{\frac{1}{3}\sin(3x)}}_{y}$$

✓

14. Suppose $e^{xy} = \cos(x)$. What is $\frac{dy}{dx}$ (as a function of x and y)?

LHS: $\frac{d}{dx} e^{xy} = (xy' + y) e^{xy}$

RHS: $= \frac{d}{dx} \cos(x) = -\sin(x)$

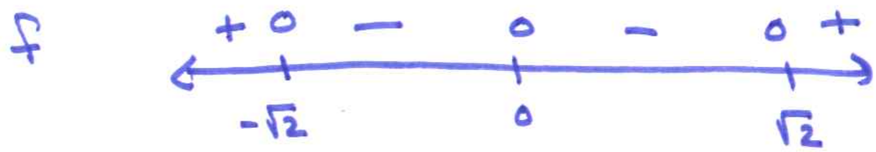
so $y' x e^{xy} = -\sin(x) - y e^{xy}$

so $\boxed{y' = -\left(\frac{\sin(x) + y e^{xy}}{x e^{xy}}\right)}$

15. Let $f(x) = x^4 - 2x^2$.

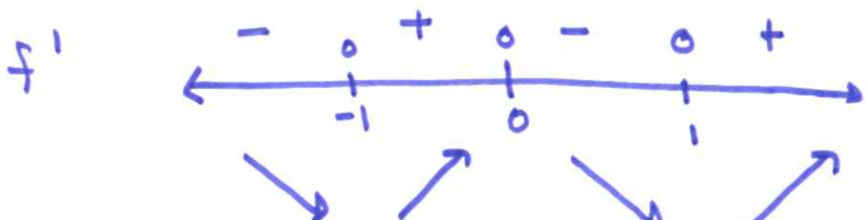
(a) For what intervals of x is $f(x)$ positive? negative?
For what values of x is $f(x)$ equal to 0?

$$x^4 - 2x^2 = x^2(x^2-2) = x^2(x+\sqrt{2})(x-\sqrt{2})$$



(b) For what intervals of x is $f(x)$ increasing? decreasing?

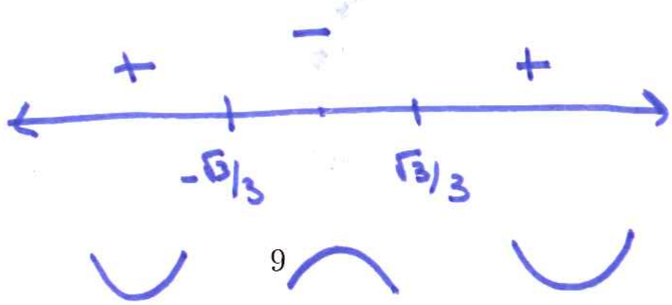
$$f' = 4x^3 - 4x = 4x(x+1)(x-1)$$



(c) For what intervals of x is $f(x)$ concave up? concave down?

$$f'' = 12x^2 - 4 = 12(x^2 - 1/3)$$

$$= 12(x + \sqrt{3}/3)(x - \sqrt{3}/3)$$



(d) Sketch a detailed graph of $f(x) = x^4 - 2x^2$ on the grid below.
Label inflection points if any.
Label your y-axis to give scale.