MATH 8: Practice Midterm II
November 10, 2004

Show all your work. Full credit may not be given for correct answers if they are not adequately justified. Good luck!

1. Find a power series that converges to each of the following functions and give the radius and interval of convergence. (Do this by manipulating geometric series, **not** by Taylor's formula.)

   (a) $\ln(1+x)$.

   (b) $\dfrac{3}{27-x^3}$.

2. Suppose you have a function $f(x)$ such that $f(x)$'s third Taylor polynomial at $x=1$ is $P_3(x) = 1-(1/2)(x-1)+(x-1)^2+(2/3)(x-1)^3$, and assume that **all** of $f(x)$'s derivatives satisfy $\left|\frac{d^n f}{dx^n}\right| \leq 5$ on the interval $(0,2)$.

   (a) Given the above data, approximate $f(1.5)$.
   (b) Bound the difference $|f(1.5) - P_3(1.5)|$ using the above data, and justify your answer.
   (c) Given the above data, can you determine $f(x)$'s second derivative at $x=1$? If so find it, if not why.

3. Find the distance between the point $P = (-25, -13, 8)$ and the plane with equation $3x + y - z = 3$.

4. Find the line of intersection of the planes $x+y+z=3$ and $x+2y+3z=6$.

5. Suppose $\vec{u}$ and $\vec{v}$ are in the plane containing the origin determined by $3x+2y+z=0$ and that that $\vec{u}$, $\vec{v}$, and $\vec{w}$ satisfy $\vec{v}\cdot(\vec{w}\times\vec{u}) = 0$. What is the equation of a plane through the origin that contians $\vec{u}$ and $\vec{w}$? Why?

6. Suppose $\vec{u}(3) =< 1,1,2 >$, $\vec{v}(3) =< 3,1,-1 >$, $\frac{d\vec{u}}{dt}(3) =< -1,0,2 >$ and $\frac{d\vec{v}}{dt}(3) =< 0,-2,3 >$.

   (a) Compute $\frac{d}{dt}[\vec{u}\cdot\vec{v}]$ at $t=3$.
   (b) Compute $\frac{d}{dt}[\vec{u}\times\vec{v}]$ at $t=3$.
   (c) Compute $\frac{d}{dt}[e^t\vec{u}]$ at $t=3$.

7. Let $\vec{r}(t) =< \sin(t)+t, \cos(t), 3 >$.

   (a) Find the tangent line to the curve given by $\vec{r}(t)$ at $t=\frac{\pi}{4}$.
   (b) Find the legnth of this curve for $0 \leq t \leq 1$. (Hint: $1+\cos(2\theta) = 2\cos(\theta)^2$).

(c) Sketch the curve (Challenging).

8. Find the third degree Taylor polynomial for $\tan(x)$ about $a = \frac{\pi}{4}$.

9. Suppose we have a plane containing the points $(1, 1, 0)$, $(2, 1, 3)$ and $(1, 0, 5)$, and a line determined by $\frac{x-2}{2} = \frac{y-3}{5} = z - 1$.

   (a) Find an equation for the plane.

   (b) Do the plane and line intersect? If so find the points of intersection.

10. Find the vector projection and the scalar projection (i.e., component) of $\vec{b}$ on $\vec{a}$ where $\vec{b} =< 2, 1, 4 >$ and $\vec{a} =< 1, 2, 3 >$.