7.4 Derivatives of Logarithmic Functions

6) Differentiate

$$f(x) = \log_{10}\left(\frac{x}{x-1}\right)$$

ANSWER $\log_{10}\left(\frac{x}{x-1}\right) = \dfrac{\ln(x) - \ln(x-1)}{\ln 10}$

$$\therefore \quad f'(x) = \frac{1}{\ln 10}\left(\frac{1}{x} - \frac{1}{x-1}\right)$$

10) Differentiate

$$f(t) = \frac{1+\ln t}{1-\ln t}$$

ANSWER USE THE QUOTIENT RULE:

$$f'(t) = \frac{(1-\ln t)\frac{1}{t} - (1+\ln t)\left(-\frac{1}{t}\right)}{(1-\ln t)^2}$$

$$= \frac{1-(\ln t)^2}{t(1-\ln t)^2}$$

28) Find y' & y''

$$y = \ln(\sec x + \tan x)$$

ANSWER

USE THE CHAIN RULE

$$y' = \frac{1}{\sec x + \tan x}\left(\sec x \tan x + \sec^2 x\right)$$

$$= \frac{1}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}\left(\frac{\sin x}{\cos^2 x} + \frac{1}{\cos^2 x}\right)$$

$$= \frac{1}{\left(\frac{1+\sin x}{\cos x}\right)} \cdot \left(\frac{1+\sin x}{\cos^2 x}\right) = \frac{1}{\cos x} = \sec x$$

$$\therefore y'' = \sec x \tan x$$

32) Differentiate f & find the domain

$$f(x) = \ln \ln \ln x$$

ANSWER Use the chain rule

$$f'(x) = \frac{1}{\ln \ln x}(\ln \ln x)' = \frac{1}{\ln \ln x}\,\frac{1}{\ln x}\,(\ln x)' = \frac{1}{\ln \ln x}\,\frac{1}{\ln x}\,\frac{1}{x}$$

and for the domain $\ln \ln x > 0 \Rightarrow \ln x > 1$

$\Rightarrow x > e$ $\quad\therefore$ we have domain $= (e, \infty)$

7.5 Inverse Trigonometric Functions

46) Find $\lim_{x \to 0^+} \tan^{-1}(\ln x)$.

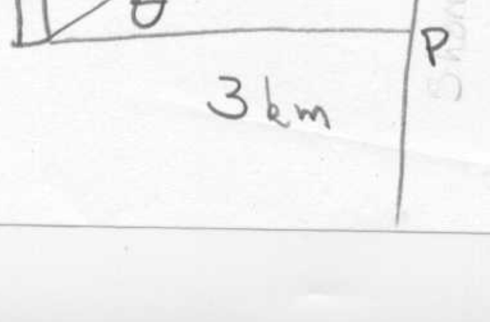
ANSWER

By substituting $y = \ln x$ we have

$$\lim_{x \to 0^+} \tan^{-1}(\ln x) = \lim_{y \to -\infty} \tan^{-1}(y) = -\pi/2$$

50) A lighthouse is located on a small island, 3 km away from the nearest point P on a straight shoreline, and its light makes 4 revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

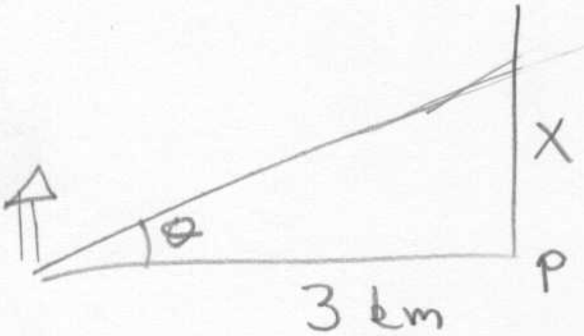
ANSWER



Let x be the distance from P to where the light is. We want to calculate

$$\left.\frac{dx}{dt}\right|_{x = 1\text{ km}}.$$

7.5 50) (continued)



Since the light makes 4 revolutions per minute

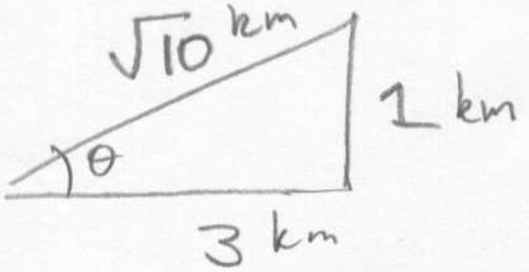
$$\theta(t) = \frac{2\pi t}{4 \text{ minutes}}, \quad \therefore \quad \frac{d\theta}{dt} = \frac{\pi}{2 \text{ minutes}}$$

& by the triangle we have

$$\tan\theta = \frac{x}{3}$$

so $$\frac{dx}{dt} = 3\sec^2\theta \, \frac{d\theta}{dt}$$

Now when $x = 1$ km, we have the triangle



so $\sec\theta = \frac{\sqrt{10}}{3}$ km $\therefore$ $\sec^2\theta = \frac{10}{9}$ km

so $$\left.\frac{dx}{dt}\right|_{x=1} = 3 \cdot \frac{10}{9} \cdot \frac{\pi}{2} \text{ km/minutes} = \frac{5\pi}{3} \text{ km/minutes}$$

7.6 Hyperbolic Functions

4) Find the numerical value.

a) $\cosh 3$ b) $\cosh(\ln 3)$

ANSWER

a) $\cosh 3 = \dfrac{e^3 + e^{-3}}{2} = \dfrac{e^6 + 1}{2e^3}$

b) $\cosh(\ln 3) = \dfrac{e^{\ln 3} + e^{-\ln 3}}{2} = \dfrac{3 + \frac{1}{3}}{2}$

$= \dfrac{5}{3}.$

6) Find the numerical value

a) $\sinh 1$ b) $\sinh^{-1} 1$

ANSWER

a) $\sinh 1 = \dfrac{e^1 - e^{-1}}{2} = \dfrac{e - \frac{1}{e}}{2} = \dfrac{e^2 - 1}{2e}$

b) see next page

6) b) we want to solve for x where

$$x = \sinh^{-1} 1$$

i.e. $\sinh(x) = 1$

$$\therefore \frac{e^x - e^{-x}}{2} = 1$$

$$\Rightarrow e^x - e^{-x} = 2$$

Now let $x = \ln y$

we have $e^{\ln y} - e^{-\ln y} = 2$

$$\Rightarrow y - \frac{1}{y} = 2$$

$$\Rightarrow y^2 - 1 = 2y$$

$$\Rightarrow y^2 - 2y - 1 = 0$$

Now use the quadratic formula

$$\Rightarrow y = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$\therefore x = \ln(1 \pm \sqrt{2})$ but domain of $\ln y$ is $y \geq 0$

$\therefore x = \ln(1 + \sqrt{2})$.

7.6 (continued)

34) Find the derivative of

$$F(x) = \sinh x \tanh x$$

ANSWER Use the product rule:

$$F'(x) = \sinh x(\operatorname{sech}^2 x) + \tanh x(\cosh x)$$

$$= \operatorname{sech} x \tanh x + \sinh x$$