Chapter 9.1 Core Problems (Evens)

#2 Let $f(x) = \sqrt{4-x^2}$ for $0 \le x \le 2$. then

$$f'(x) = \frac{-x}{\sqrt{4-x^2}}, \quad f'(x)^2 = \frac{x^2}{4-x^2} \text{ and}$$

$$1 + f'(x)^2 = \frac{4-x^2}{4-x^2} + \frac{x^2}{4-x^2} = \frac{4}{4-x^2}.$$

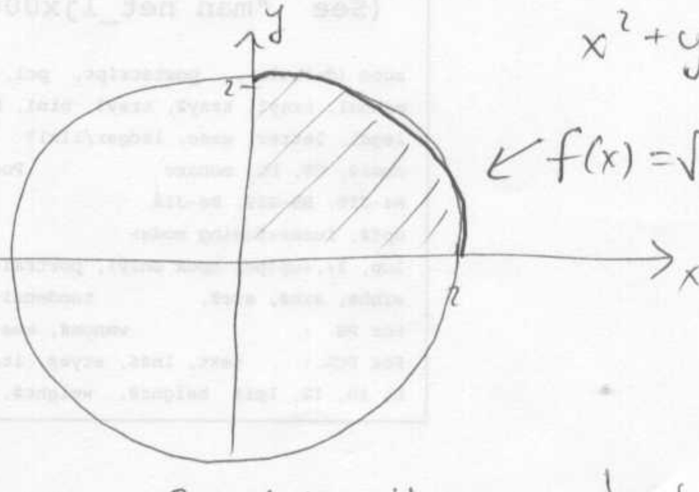
So the arc length is

$$L = \int_0^2 \sqrt{1+f'(x)^2}\,dx = \int_0^2 \frac{2}{\sqrt{4-x^2}}\,dx$$

$$= 2 \cdot \arcsin\left(\frac{x}{2}\right)\Big|_0^2 = 2(\arcsin(1) - \arcsin(0))$$

$$= 2\left(\frac{\pi}{2} - 0\right) = \boxed{\pi}$$

This makes sense since $f(x)$ describes the first quadrant of a circle of radius 2:



$x^2 + y^2 = 2^2$

$\leftarrow f(x) = \sqrt{4-x^2}, \quad 0 \le x \le 2$

So circumference $= 2\pi(2) = 4\pi$, so $\frac{1}{4} \cdot$ circumference $= \pi$!

24 The arc-length of $f(x) = x\ln x$ for the interval $1 \le x \le 3$ is given by

$$L = \int_1^3 \sqrt{1 + f'(x)^2}\,dx = \int_1^3 \sqrt{1 + (1+\ln x)^2}\,dx$$

and we'll call $g(x) = \sqrt{1 + (1+\ln(x))^2}$.

Now Simpson's Rule with $n = 10$ and $\Delta x = (3-1)/10$ gives us

$$L = \int_1^3 g(x)\,dx \approx \frac{\Delta x}{3}\Big(g(1) + 4g(1.2) + 2g(1.4) + \cdots + 2g(2.6) + 4g(2.8) + g(3)\Big)$$

$$\approx \frac{1}{15}\big(1.414 + 6.194 + 3.338 + 7.111 + 3.753 + 7.865 + 4.098 + 8.501 + 4.393 + 9.050 + 2.324\big)$$

$$\approx 3.869.$$

Now maple tells me that in fact

$$L = \int_1^3 \sqrt{1 + (1+\ln x)^2}\,dx \approx 3.869616\ldots$$

So we're pretty close.

Math 104 Section 9.2
Even Core Problems

2. The surface area of $y = \sin^2 x$ for $0 \le x \le \frac{\pi}{2}$ revolved about the x-axis is given by

$$S = \int_0^{\frac{\pi}{2}} 2\pi \sin^2 x \sqrt{1 + 4\sin^2 x \cdot \cos^2 x}\, dx$$

noting that $y' = 2\sin x \cdot \cos x$.

20 Let $f(x) = \sqrt{1+e^x}$. Then

$f'(x) = \frac{1}{2}(1+e^x)^{-1/2} \cdot e^x$ so that

$f'(x)^2 = \frac{1}{4} \frac{e^{2x}}{1+e^x}$ and so

$$1 + f'(x)^2 = \frac{1}{4}\frac{4+4e^x}{1+e^x} + \frac{1}{4}\frac{e^{2x}}{1+e^x} = \frac{1}{4}\frac{e^{2x}+4e^x+4}{1+e^x}$$ and

$$\sqrt{1+f'(x)^2} = \frac{1}{2}\sqrt{\frac{e^{2x}+4e^x+4}{1+e^x}} = \frac{1}{2}\sqrt{\frac{(e^x+2)^2}{1+e^x}} = \frac{1}{2}\frac{e^x+2}{\sqrt{1+e^x}}$$

So that the surface area given by revolving $f(x)$ around the x-axis from $0 \le x \le 1$ is given by

$$S = \int_0^1 2\pi\sqrt{1+e^x} \cdot \frac{1}{2}\frac{e^x+2}{\sqrt{1+e^x}}\,dx = \pi\int_0^1 (e^x+2)\,dx = \pi(e^x+2x)\Big|_0^1$$

$$= \pi(e+2-(1+0)) = \pi(e+1).$$

Forget Simpson's Rule!!!