

MATH 170 IDEAS IN MATHEMATICS (SUMMER 2006)

Problem Set 1: First Introduction to Formal Systems

Due in class Thursday, May 18th

1. Playing with the MIU-system

- a. Write out formal derivations of the following strings:
MIU, MIIU, MIIUIU, MUUII, MUUIUIIU.
- b. Find a derivation for the theorem MUIIU “different” from the one Hofstadter gives on page 36. Can you find a derivation using only rules II and III? How many possible derivations can you find for MUIIU? Are there finitely many?
- c. Find a derivation for the string MUUII. Now starting from the string MUUII, write down the “decision tree”, as in page 40, and go three levels deep. How many possible different theorems are derivable from MUUII (including itself) in three steps? Do the same for the string MIIUU.

2. Thinking about the MIU-system

- a. Does the MIU-system have arbitrarily long theorems?
- b. Once a theorem ends in U, must any theorem derivable from it also end in U?
- c. Say a theorem x of the MIU-system is *reduced* if no shorter theorem can be derived from it. For example, MIUUI is not reduced. Which of the theorems in part 1a above are reduced? Can you give a procedure to decide if a theorem is reduced? What do all the reduced “look like”? How do you know your procedure will work? Is the string MU reduced, and will your procedure work in this case?
- d. Make a “backwards decision tree” for the string MU: Write the string MU. then above it, list all possible ways you could get to MU from other possible strings using one rule (these are MUUU, MIII, and M), etc. Do this for three levels. Can you say anything in general about what kind of string would be needed to give rise to MU?
- *e. Is MU a theorem?