# MATH 170 IDEAS IN MATHEMATICS (SUMMER 2006)
## **Problem Set 5:** Some metalogic.
Due in class Thursday, June 1st

**1. Negations**

Let $P_1, \dots, P_n$ be propositions and let $P(P_1, \dots, P_n)$ be a proposition built from $P_1, \dots, P_n$ and the logical symbols. For example,

$$P(P_1) = P_1 \vee \neg P_1, \quad P(P_1, P_2) = \neg P_1 \vee (P_1 \wedge P_2), \quad P(P_1, P_2, P_3) = (P_1 \wedge P_2) \vee P_3,$$

are possible such propositions. You should think of $P$ as a function and $P_1, \dots, P_n$ as variables. For example, if $P(P_1, P_2) = P_1 \vee P_2$ then

$$P(P_1, P_1 \wedge P_2) = P_1 \vee (P_1 \wedge P_2) \quad \text{and} \quad P(Q \vee (\neg R), S) = (Q \vee (\neg R)) \vee S,$$

if $Q, R, S$ are more propositions.

Now prove (or explain why it's true) the following metalogical statement from class:

**Proposition 1.** *Let $P(P_1, \dots, P_n)$ and $Q(P_1, \dots, P_n)$ be propositions built from propositions $P_1, \dots, P_n$ as above. Then*

$$P(P_1, \dots, P_n) \equiv Q(P_1, \dots, P_n) \quad \textit{if and only if} \quad P(\neg P_1, \dots, \neg P_n) \equiv Q(\neg P_1, \dots, \neg P_n).$$

You should not have to draw any truth tables, but you might want to make general statements about truth tables.

For example, if $P(P_1, P_2) = \neg(P_1 \wedge P_2)$ and $Q(P_1, P_2) = (\neg P_1) \vee (\neg P_2)$ then the metalogical statement

$$P(P_1, P_2) \equiv Q(P_1, P_2) \quad \text{if and only if} \quad P(\neg P_1, \neg P_2) \equiv Q(\neg P_1, \neg P_2),$$

in symbols just says

$$\neg(P_1 \wedge P_2) \equiv (\neg P_1) \vee (\neg P_2) \quad \text{if and only if} \quad \neg(\neg P_1 \wedge \neg P_2) \equiv (\neg(\neg P_1)) \vee (\neg(\neg P_2)).$$

So since you know the left hand side is true, then so is the right hand side, though you could have also proven the right hand side from scratch using truth tables.

**2. Metametalogic?**

**a.** Discuss the meta-interrelationship between the logical symbols, the symbol $\equiv$, and the usage of *if and only if*. If the propositions, logical symbols, and all propositions built from them make up what we call *symbolic logic*, then is the notion of logically equivalent (the symbol $\equiv$) a part of symbolic logic, or a part of symbolic *meta*logic? Where does *if and only if* fit in?

**b.** Can you imagine adding $\equiv$ as a logical symbol? If so what would it's truth table be? Can you write it in terms of the other logical symbols, i.e. can you find a proposition $P(P_1, P_2)$ such that

(1) $$P(P_1, P_2) \quad \equiv \quad \text{“}P_1 \equiv P_2\text{”}$$

for any two propositions $P_1$ and $P_2$, and where I'm think of "$P_1 \equiv P_2$" as its own proposition? What "type" of statement is line (1) above: a logical statement, a metalogical statement, a metametalogical statement, or perhaps a "strange loop" statement?