UNIVERSITY OF PENNSYLVANIA DEPARTMENT OF MATHEMATICS
Math 170 Ideas in Mathematics Summer Session I 2007
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Homework #1 (due Tuesday 29 May 2007)

1. Tally scheme. Create your own 5 or 10 step tally scheme.

2. Some ancient fractions. Writing fractions in the various ancient number systems was oftentimes quite a challenge!

- *a)* Write the numbers $\frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}, \frac{2}{7}, \frac{2}{8}, \frac{2}{9}, \frac{2}{10}, \frac{2}{11}, \frac{2}{15}$ in ancient Egyptian. Hint: you should put the fractions in reduced form before beginning.
- *b)* Write the numbers $\frac{4}{5}, \frac{4}{6}, \frac{4}{7}, \frac{4}{8}, \frac{4}{9}, \frac{4}{10}, \frac{4}{11}$ in ancient Egyptian.
- *c)* Write the numbers $\frac{2}{15}$ in ancient Egyptian in three different ways.
- *d)* Can you write $\frac{1}{5}$ in ancient Roman? Explain.
- *e)* Explain why the ancient Mayan symbols for the numbers 10 and 105 might be confused. List three other pairs of numbers that might be confused. Find a set of three numbers that might be mutually confused.

3. Digital roots and remainders. The *digital root* of a number written in a based positional numbering system is gotten by successively adding up its digits (now ignoring their positions) until you get something in the 1's place. For example, in our numbering system, to take the digital root of 489 we proceed as follows, at each step we add up the digits of the number in the last step:

$$489 \quad \longrightarrow \quad 21 \quad \longrightarrow \quad 3$$

so the digital root of 489 is 3. Extracting the digital root of 154 proceeds as

$$154 \quad \longrightarrow \quad 10 \quad \longrightarrow \quad 1.$$

- *a)* For the numbers from 1 to 30, extract their digital roots in our numbering system. Do you notice anything? In particular, what can you say about the numbers with digital root 9? Test your hypothesis on a few large numbers.
- *b)* What do you think is the corresponding phenomenon in the ancient Mayan system? It might help to want to write out the digital roots for some Mayan numbers. What can you say about the numbers with Mayan digital root 19?
- *c)* Find the remainder of 446 when divided by 19 in two different ways: using ideas from part *b)*, and using long division. Which is faster?

4. FAPP In For All Practical Purposes (FAPP):
- *a)* Ed. 6, Chapter 9, exercises 1, 3a, 18, 39, 46, 58. *or*
  Ed. 7, Chapter 16, exercises 47, 49a, 14, 70, 74.
- *b)* On-line FAPP Chapter 16 "self-test" available at `www.whfreeman.com/fapp7e/`.
- *c)* Ed. 6, Chapter 10, exercises 2, 4, 14, 15 ,16, 17. *or*
  Ed. 7, Chapter 17, exercises 2, 4, 35, 38, 39, 40.