Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Fall 2011

Problem Set # 4 (due Friday 23 September 2011)

Recall: If $\gamma : \mathbb{R} \to \mathbb{R}^2$ is a parameterized curve in the x-y-plane given by $\gamma(t) = (\gamma_1(t), \gamma_2(t))$, and $f : \mathbb{R}^2 \to \mathbb{R}$ is a function, then the *lift* of γ to the graph of f is a new parameterized curve $\alpha : \mathbb{R} \to \mathbb{R}^3$ in 3-space defined by $\alpha(t) = (\gamma_1(t), \gamma_2(t), f(\gamma_1(t), \gamma_2(t)))$.

In CM 17.2, there's a formula for the length of a segment of a parameterized curve. If $\beta : \mathbb{R} \to \mathbb{R}^n$ is any parameterized curve in n-space, and if $a \leq b$ are real numbers, then we have:

$$(\text{length of } \beta \text{ from } t = a \text{ to } t = b) = \int_a^b \|\vec{\beta}'(t)\| \, dt$$

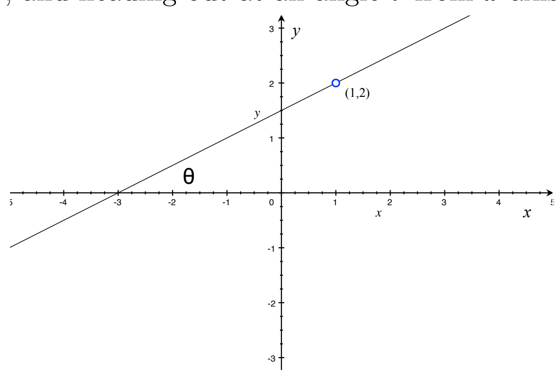
Since for each t, $\|\beta(t)\|$ is a number, the integral above is just a standard single-variable definite integral.

Reading: CM 17.1-3

1. Let P be a point in $\mathbb{R}^3$ and let $\vec{v}$ be a direction vector at P. Find a parameterization of the line through P in the direction $\vec{v}$ and with constant speed 1. (Hint: Look at the section "Unit vectors" in chapter 13.1, page 692.) How many other parameterizations of this line exist with constant speed 1?

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = xy$. Let $P = (1, 2, 2)$.

a) For each angle θ from 0 to 2π, find a parameterization $\gamma_\theta : \mathbb{R} \to \mathbb{R}^2$ for the line starting at $(1, 2)$ in the x-y-plane at time $t = 0$, and heading out at an angle θ from x-axis with constant speed 1.



b) For each θ, let α_θ be the lift of your γ_θ to the graph of f. Write $\alpha_\theta(t)$.

c) As a function of θ, calculate the length of α_θ from $t = 0$ to $t = 1$. This is the length travelled on the graph of f in one time unit walking at a compass angle θ.

d) For which θ is the length of this path maximal/minimal? Only set up/explain what you need to do! You don't need to evaluate anything!
(For **extra credit**: actually find the θs giving maximal/minimal length travelled! You can even use a computer!)

e) Find the direction vectors (in the plane) you have to start walking in from P to achieve the greatest ascent and descent on the graph (use the gradient). Calculate the angle this vector makes with the x-axis (i.e. with the vector $\vec{\imath}$).
(For **extra credit**, continue: how do these angles compare with what you computed above? Should they necessarily be the same?)

3. CM 17.1 Problems 44, 48, 68

4. CM 17.2 Problems 37

5. CM 17.3 Problem 21-28 (you don't need to explain your answers).

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