Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Fall 2011

Problem Set # 5 (due Friday 7 October 2011)

Recall: Let $a \leq b$ be real numbers, $[a,b]$ the closed interval from a to b in $\mathbb{R}$, $\gamma : [a,b] \to \mathbb{R}^2$ a parameterized curve, and $\vec{F}$ a vector field in $\mathbb{R}^2$, then the line integral of $\vec{F}$ along γ is computed by the definite integral

$$\int_\gamma \vec{F} = \int_a^b \vec{F}(\gamma(t)) \cdot \gamma'(t)\, dt.$$

In CM, they like to call parameterized curves $\vec{r}$, so they write

$$\int_C \vec{F} \cdot d\vec{r}$$

for the line integral of $\vec{F}$ along the curve C which is the image of $\vec{r}$ from $t = a$ to $t = b$. I personally do not prefer this notation but it's good to get used to both. There is even a thrid commonly used notation explained in CM on page 938.

Reading: CM 17.4, 18.1-2.

1. CM 17.4 Problem 18 (redraw the pictures and put the arrows indicating flow direction)

2. CM 18.1 Exercises 2, 4, 6, 12, 16
Problem 38

3. CM 18.2 Exercises 4, 12, 16, 20
Problems 30, 34

4. (Extra credit) Define a vector field by

$$\vec{F}(x,y) = \begin{cases} -\frac{y}{|y|}\vec{\imath} & \text{if } y \neq 0 \\ \vec{0} & \text{if } y = 0 \end{cases}$$

Parameterize the following closed curves and calculate the (circulation) line integral of $\vec{F}$ along them:

a) A circle of radius 1 about the origin (going counter clockwise).
b) A circle of radius 1 about $(0, \frac{1}{2})$ (going counter clockwise).
c) A circle of radius 1 about $(0, \frac{\sqrt{2}}{2})$ (going counter clockwise).
d) A circle of radius 1 about $(0, \frac{\sqrt{3}}{2})$ (going counter clockwise).
e) A circle of radius 1 about $(0, 1)$ (going counter clockwise).

Explain in words what is happening.

Emory University, Department of Mathematics & CS, 400 Dowman Dr NE W401, Atlanta, GA 30322
E-mail address: auel@mathcs.emory.edu