Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Spring 2010

Problem Set # 1 (due Friday 22 Jan 2010)

If S is a set of elements, then $s \in S$ means "s is an element of the set S." If T is another set, then $T \subset S$ means "every element of T is an element of S" or "T is a *subset* of S."

Recall that a *multivariable mapping* $f : \mathbb{R}^n \to \mathbb{R}^m$ is a recipe, which given any element $v \in \mathbb{R}^n$, produces an element $f(v) \in \mathbb{R}^m$. For certain cases of n and m such mappings have other names:

- $n = 1$ and $m = 1$, called *single variable function*,
- $n = 1$ and $m > 1$, called *parametrized curve (in* $\mathbb{R}^m$*)*,
- $n > 1$ and $m > 1$, called *vector field*,
- $n \geq 1$ and $m = 1$, called *function (on* $\mathbb{R}^n$*)*.

More generally, given subsets $V \subset \mathbb{R}^n$ and $W \subset \mathbb{R}^m$, a (multivariable) mapping $f : V \to W$ is a recipe, which given any element $v \in V$ produces an element $f(v) \in W$. We call V the *domain* and W the *codomain* of f. The *image* $\mathrm{im}(f) \subset W$ of f is the subset of the codomain consisting of all elements $w \in W$ such that $w = f(v)$ for some $v \in V$ in the domain.

Definition. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a multivariable mapping, then the *graph* of f is the multivariable mapping $\Gamma_f : \mathbb{R}^n \to \mathbb{R}^{n+m}$ defined by $\Gamma_f(v) = (v, f(v)) \in \mathbb{R}^n \times \mathbb{R}^m = \mathbb{R}^{n+m}$, i.e. writing $v = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and $f(v) = (f_1(v), \ldots, f_m(v)) \in \mathbb{R}^m$, then $\Gamma_f(x_1, \ldots, x_n) = (x_1, \ldots, x_n, f_1(v), \ldots, f_m(v))$.

1. *Graphs of Multivariable Functions*

a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function on $\mathbb{R}^2$ and let $\Gamma_f : \mathbb{R}^2 \to \mathbb{R}^3$ be its graph. Give $\mathbb{R}^3$ the standard (x, y, z) coordinates. State an analogy of the "vertical line test" for images $\mathrm{im}(\Gamma_f)$ of graphs of functions f on $\mathbb{R}^2$.

b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function on $\mathbb{R}^2$. Give $\mathbb{R}^3$ the standard (x, y, z) coordinates. Describe the intersection, in the x-y-plane, of $\mathrm{im}(\Gamma_f)$ with the x-y-plane, in terms of the function f. What other names have we given this set?

c) Now let $f(x, y) = x^2 + y$. Draw, in the x-z-plane (where x is horizontal), the intersection of $\mathrm{im}(\Gamma_f)$ with the x-z-plane. Find a function $g : \mathbb{R} \to \mathbb{R}$ so that $\mathrm{im}(\Gamma_g)$, thought of in the x-z-plane, is this intersection.

d) Again, let $f(x, y) = x^2 + y$. For each $\theta \in [0, \pi]$, let P_θ be the plane through the z-axis and the point $(\cos\theta, \sin\theta, 0)$. Describe, in the plane P_θ what the intersection of P_θ and $\mathrm{im}(\Gamma_f)$ can look like for varying θ. Orienting the plane P_θ with the z-axis vertically, find a function $g_\theta : \mathbb{R} \to \mathbb{R}$ whose graph is this intersection.

2. CM Problem 12.1.29

3. CM Problems 12.2.15 and 12.2.24

4. CM Exercise 12.3.14 and Problem 12.3.33

5. CM Problem 12.5.30

6. CM Problem 13.1.33

7. CM Problems 13.3.37, 13.3.39, and 13.3.40

8. CM Problems 13.4.25 and 13.4.28

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