Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Spring 2010

Problem Set # 5 (due Wednesday 3 March 2010)

Recall: Let $a \leq b$ be real numbers, $[a,b] \subset \mathbb{R}$ the closed interval from a to b, $\gamma : [a,b] \to \mathbb{R}^2$ a parameterized curve, and $\vec{F}$ a vector field in $\mathbb{R}^2$, then the line integral of $\vec{F}$ along γ is computed by the definite integral

$$\int_\gamma \vec{F} = \int_a^b \vec{F}(\gamma(t)) \cdot \gamma'(t)\, dt.$$

In CM, they like to call parameterized curves $\vec{r}$, so they write

$$\int_C \vec{F} \cdot d\vec{r}$$

for the line integral of $\vec{F}$ along the curve C which is the image of $\vec{r}$ from $t = a$ to $t = b$. I personally do not prefer this notation.

Reading: CM 17.4, 18.1-2.

1. CM 17.4 Problem 17

2. CM 18.1 Exercises 2, 4, 6, 12, 14
Problem 38

3. CM 18.2 Exercises 12, 16, 20
Problems 30, 34

4. * Define a vector field by

$$\vec{F}(x,y) = \begin{cases} -\frac{y}{|y|}\vec{\imath} & \text{if } y \neq 0 \\ \vec{0} & \text{if } y = 0 \end{cases}$$

Parameterize the following closed curves and calculate the (circulation) line integral of $\vec{F}$ along them:

a) A circle of radius 1 about the origin (going counter clockwise).
b) A circle of radius 1 about $(0, \frac{1}{2})$ (going counter clockwise).
c) A circle of radius 1 about $(0, \frac{\sqrt{2}}{2})$ (going counter clockwise).
d) A circle of radius 1 about $(0, \frac{\sqrt{3}}{2})$ (going counter clockwise).
e) A circle of radius 1 about $(0, 1)$ (going counter clockwise).

Explain in words what is happening.

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