

Problem Set # 1 (due Friday 27 Jan 2012)

Notations: If S is a set of elements (points, numbers, elephants, ...) then the notation “ $s \in S$ ” means “ s is an element of the set S .” If T is another set, then the notation “ $T \subset S$ ” means “every element of T is an element of S ” or “ T is a *subset* of S .” For example, the set of squares is a subset of the set of rectangles.

Recall that we have notations for the following sets:

- \mathbb{R} is the real 1-dimensional “number line”
- \mathbb{R}^2 is the real 2-dimensional “Cartesian plane”
- \mathbb{R}^3 is the real 3-space
- \mathbb{R}^n is the real n -space.

We can express a point $v \in \mathbb{R}^n$ as an n -tuple (x_1, \dots, x_n) of real numbers.

A *multivariable mapping* $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a recipe, which given any element $v \in \mathbb{R}^n$, produces an element $f(v) \in \mathbb{R}^m$. For certain cases of n and m such mappings have other names:

- $n = 1$ and $m = 1$, called *single variable function*,
- $n = 1$ and $m > 1$, called *parametrized curve (in \mathbb{R}^m)*,
- $n \geq 1$ and $m = 1$, called *function (on \mathbb{R}^n)*.

More generally, given subsets $V \subset \mathbb{R}^n$ and $W \subset \mathbb{R}^m$, a (multivariable) mapping $f : V \rightarrow W$ is a recipe, which given any element $v \in V$ produces an element $f(v) \in W$. We call V the *domain* and W the *codomain* of f . The *image* $\text{im}(f) \subset W$ of f is the subset of the codomain consisting of all elements “hit by f ” i.e. all elements $w \in W$ such that $w = f(v)$ for some $v \in V$ in the domain.

Definition. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a multivariable mapping, then the *graph* of f is the subset $\Gamma_f \subset \mathbb{R}^{n+m}$ consisting of all points $(v, f(v)) \in \mathbb{R}^{n+m}$ for $v \in \mathbb{R}^n$, i.e. all points $(x_1, \dots, x_n, f_1(x), \dots, f_m(x))$ where we write $v = (x_1, \dots, x_n) \in \mathbb{R}^n$ and $f(v) = (f_1(v), \dots, f_m(v)) \in \mathbb{R}^m$.

1. Graphs of Multivariable Functions

- a) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function on \mathbb{R}^2 . Give \mathbb{R}^3 the standard (x, y, z) coordinates. Describe the intersection of Γ_f with the x - y -plane by an implicit equation in terms of the function f . Draw this set for $f(x, y) = x^2 - xy$.
- b) Now let $f(x, y) = x^2 + y$. Draw the intersection of Γ_f with the x - z -plane and find both an implicit equation and a parameterization describing it.

2. CM Problem 12.1.29 (it turns out to be a surface in \mathbb{R}^3 , write an implicit equation for it)

3. CM Problems 12.2.15 and 12.2.24

4. CM Exercise 12.3.14 and Problem 12.3.33

5. CM Problem 12.5.30

6. CM Problem 13.1.33

7. CM Problems 13.3.37, 13.3.39, and 13.3.40

8. CM Problem 13.4.25