Problem Set # 3 (due Friday 10 Febraury 2012)

Gradients. If $f : \mathbb{R}^2 \to \mathbb{R}$ is a (2-variable) function and $P \in \mathbb{R}^2$ is a point, then the **gradient** of f at P is the vector

$$\nabla f\big|_P = \frac{\partial f}{\partial x}\Big|_P \vec{\imath} + \frac{\partial f}{\partial y}\Big|_P \vec{\jmath}$$

at P. It points in the direction of greatest instantaneous increase of f at P and has magnitude equal to the greatest rate of increase.

If $f : \mathbb{R}^3 \to \mathbb{R}$ is a (3-variable) function and $P \in \mathbb{R}^3$ is a point, then the **gradient** of f at P is the vector

$$\nabla f\big|_P = \frac{\partial f}{\partial x}\Big|_P \vec{\imath} + \frac{\partial f}{\partial y}\Big|_P \vec{\jmath} + \frac{\partial f}{\partial z}\Big|_P \vec{k}$$

at P. It points in the direction of greatest instantaneous increase of f at P and has magnitude equal to the greatest rate of increase.

Normal vectors to tangent planes of surfaces. We have two ways of describing surfaces in $\mathbb{R}^3$: as graphs of 2-variable functions (this is a parametric description) and as level surfaces of 3-variable functions (this is an implicit description).

If $f : \mathbb{R}^2 \to \mathbb{R}$ is a (2-variable) function and $(a, b) \in \mathbb{R}^2$ is a point, then the vector

$$\vec{n} = \nabla f\big|_{(a,b)} - \vec{k} = \frac{\partial f}{\partial x}\Big|_P \vec{\imath} + \frac{\partial f}{\partial y}\Big|_P \vec{\jmath} - \vec{k}$$

is normal to the tangent plane of the graph Γ_f at the point $(a, b, f(a, b))$, provided that $\nabla f\big|_{(a,b)} \neq \vec{0}$.

If $f : \mathbb{R}^3 \to \mathbb{R}$ is a (3-variable) function and $P \in \mathbb{R}^3$ is a point such that $f(P) = 0$, then the vector

$$\vec{n} = \nabla f\big|_P$$

is normal to the tangent plane of the level surface $f(x, y, z) = 0$ at the point P, provided that $\nabla f\big|_P \neq \vec{0}$.

Reading: CM 14.5 and 17.1-3

1. CM 14.4 Exercise 13, 14, 24, 30-35 (for 30-35, no justification is required)
Problem 66, 67, 70

2. CM 14.5 Exercises 12, 18.
Problems 40 (additionally, show that for each a the tangent plane goes through the origin and that actually, the plane doesn't depend on a, what do you think is going on here?), 42, 48 (hint for part **b**: remember from single variable calculus that "rate of change" is the same as the "slope of the tangent line"), 54.

3. CM 17.1 Exercises 10, 16, 22, 27.
Problems 46, 52, 54 (hint: use problem 46), 56, 59 (parameteriza the line between the points and see if it goes through the unit sphere), 62 (parameterize the line, then for each t, calculate the distance to the origin, then use single variable calculus to find minimize the distance; hint: you can also minimize the square of the distance, then take square roots).

4. CM 17.2 Exercise 10, 12, 14 (hint: as t varies from 0 to 1, the curve sweeps out an arc on the unit circle, which you can calculate using geometry, think radians!).

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