Emory University Department of Mathematics & CS
Math 211 Multivariable Calculus
Spring 2012

Quiz # 1 St. Valuentine's Day ♡ 2012 Solutions

Directions: You had 20 minutes. No electronic devices were allowed. No notes were allowed.

1. Let S be the plane in $\mathbb{R}^3$ defined by the equation $2x + 3y + 4z = 4$.

a) Find a point on S.

Solution. I spotted the point $(x, y, z) = (0, 0, 1)$. More generally, solving for z in terms of x and y gives $z = \frac{1}{4}(4-2x-3y)$, so that for any choices of x and y, the point $(x, y, \frac{1}{4}(4-2x-3y))$ is on S.

b) Find a vector normal to S at your point.

Solution. As discussed in class, the vector $2\vec{\boldsymbol{\imath}} + 3\vec{\boldsymbol{\jmath}} + 4\vec{\boldsymbol{k}}$ (taking the coefficients of x, y, and z) will be normal to S at any point of S.

c) Find an equation for the plane parallel to S but passing through the origin.

Solution. Using the normal, call it $\vec{n}$, from part *b)* and the origin as the point, the equation $\vec{n} \cdot \vec{X} = 2x + 3y + 4z = \vec{n} \cdot \vec{0} = 0$ gives an example (since any vector dotted with the the zero vector is zero). In fact any plane written as $ax + by + cz = e$ goes through the origin if and only if $e = 0$.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = (x + 1)e^{xy+2x}$ and let Γ_f in $\mathbb{R}^3$ be the graph of f.

a) Calculate the gradient vector $\nabla f|_{(0,0)}$.

Solution. First, we compute

$$\nabla f = (e^{xy+2x} + (x + 1)(y + 2)e^{xy+2x})\vec{\boldsymbol{\imath}} + (x + 1)xe^{xy+2x}\vec{\boldsymbol{\jmath}}$$

by using the product rule, chain rule, and exponentiation rules for derivatives. Then evaluating gives $\nabla f|_{(0,0)} = (e^0 + 1 \cdot 2 \cdot e^0)\vec{\boldsymbol{\imath}} + 1 \cdot 0 \cdot e^0 = 3\vec{\boldsymbol{\imath}}$.

b) Find z such that the point $P = (0, 0, z)$ is on Γ_f.

Solution. The graph has implicit equation $z = f(x, y)$, for $(x, y) = (0, 0)$, we have that $z = f(0, 0) = 1$. So $P = (0, 0, 1)$.

c) Find an equation for the tangent plane to Γ_f at the point P from part *b)*. Write your equation in the form $ax + by + cz = d$.

Solution. The point $P = (0, 0, 1)$ on the graph is above $(0, 0)$, so by our formula from class, a normal to the tangent plane at P is

$$\nabla f|_{(0,0)} - \vec{\boldsymbol{k}} = 3\vec{\boldsymbol{\imath}} - \vec{\boldsymbol{k}}$$

and hence the point/normal formula $(\vec{n} \cdot \vec{X} = \vec{n} \cdot \vec{P})$ gives $3x - z = (3\vec{\boldsymbol{\imath}} - \vec{\boldsymbol{k}}) \cdot \vec{\boldsymbol{k}} = -1$.