Yale University Department of Mathematics
Math 225 Linear Algebra and Matrix Theory
Spring Semester 2018

Instructor:	Professor Asher Auel	**Lecture:**	LOM 206
Office:	LOM 210	**Times:**	Tue Thu 09:00 – 10:15 am
Hours:	Tue 02:15 – 04:15 pm		Tue Thu 01:00 – 02:15 pm
Text:	*Linear Algebra, 4th ed.*, Friedberg, Insel, and Spence Pearson, 2003. ISBN-13: 978-0-13-008451-4.		
Web-site:	http://math.yale.edu/~auel/courses/225s18/		
Teaching Fellow:	Joon-Hyeok Yim	**Section:**	TBA
Office:	DL 401	**Time:**	TBA

Introduction: Linear Algebra can be regarded as the art of solving linear equations. You have seen such equations since middle school: if $2x = 4$ then find x. In high school, you solve systems of 2 or 3 simultaneous linear equations in 2 or 3 variables. Such systems can be organized into a matrix equation $Ax = b$, where A is a matrix, x is a variable vector, and b is a constant vector. Linear algebra is a deep investigation into systems of simultaneous linear equations. In the course, we will examine such questions as: How do we know when a system of m linear equations in n variables has a solution? How many solutions can there be? How do we find them? More generally, we will be interested in the abstract structure of the space of solutions, which forms a vector space. Such questions are in fact fundamental to the natural sciences, computer science, economics, and statistics. Furthermore, almost all higher mathematics today (geometry, topology, number theory, analysis, differential equations, etc.) depends on linear algebra in important ways.

This course will provide a rigorous proof-based introduction to linear algebra. The main topics covered will be vector spaces, linear transformations, matrices, systems of linear equations, determinants, eigenvalues, eigenvectors, diagonalization, inner-product spaces, normal and self-adjoint operators, the spectral theorem, and applications. Time permitting, we will investigate the linear algebra behind Google's PageRank algorithm and Heisenberg's uncertainty principle in quantum mechanics. Math 225 (as opposed to Math 222) is more focused on the abstract aspects of linear algebra and will demand a great deal of maturity of mathematical thinking, not just rote problem solving. The course will try to strike a balance between computations, concepts, proofs, and applications. Problem sets will be heavily proof-based.

Grading: Your final grade will be calculated according to the table at right. Notice that more emphasis is placed on exams than on weekly homework assignments. On the other hand, completing your weekly homework will be crucial to your success on the exams.

Homework	35 %
Quizzes	10 %
Midterm (Thu 08 Mar)	20 %
Final Exam (Sun 06 May)	35 %

Exams: There will be two short in-class quizzes, announced a few days ahead. The midterm exam will take place 07:30 – 09:00 pm on Thursday 08 March in Davies Auditorium. The final exam will take place 07:00 – 10:30 pm on Sunday 06 May. Make-up quizzes and exams will only be allowed with a dean's excuse. Your top quiz scores will be weighted twice as much as your lowest score. The use of electronic devices during quizzes and exams will not be allowed.

Homework: You will be assigned a weekly problem set, due in-class on Thursday. There will be no problem sets due the week of midterm exam. The problem sets will be posted on the course web-site syllabus page the week before they are due. Your lowest problem set score will be dropped from your final grade calculation. *Late or improperly submitted homework will not be accepted.* If you know in advance that you will be unable to submit your homework at the correct time and place, you must make special arrangements ahead of time. Under extraordinary circumstances, late homework may be accepted with a dean's excuse.

Consider (as you would for any other class) the pieces of paper you turn in as a final copy: written neatly and straight across the page, on clean paper, stapled together, with nice margins, lots of space, and well organized. *If it's not readable, it won't be graded.* You should strongly consider starting with a rough draft, especially on problems requiring a proof. You will need to show your work on computational problems. You might consider taking the opportunity to learn LaTeX.

Group work, honestly: Working with other people on mathematics is not only allowable, but is highly encouraged and fun. You may work with anyone (e.g., other students in the course, students not in the course, tutors) on the rough draft of your problem sets. If done right, you'll learn the material better and more efficiently working in groups. The golden rule is:

> You may work with anyone on *solving* your homework problems,
> but you must *write* up your final draft by yourself.

Writing up the final draft is as important a process as figuring out the problems on scratch paper with your friends. Mathematical writing is very idiosyncratic—it is easy to tell if papers have been copied from others or from the internet—just don't do it! You will not learn, and will engage in academic dishonesty, by copying solutions! Also, if you work with people on a particular assignment, you *must list your collaborators at the top of the paper*, as well as any resources (e.g., Wikipedia, Wolfram Alpha) used beyond the text book. Make the process fun, transparent, and honest.

Prerequisites: Officially, the prerequisite is Math 120 Multivariable Calculus (taken earlier or concurrently). In reality, we will hardly use any calculus. However, it is important that you are comfortable with vectors and basic geometry of 3-dimensional space as taught in Math 120 (e.g., vector addition, scalar multiplication, dot product, magnitude, normal vectors, lines and planes in 3-dimensional space).

Topics covered: Subject to change.

(1) Vector spaces and subspaces. Fields. Direct sums. FIS 1.1–1.3, 5.2, Appendices C–D.
(2) Systems of linear equations. Linear dependence/independence of vectors. Basis. Dimension. FIS 1.4–1.6.
(3) Linear transformations. Null space and range. Matrix representations. Compositions of linear transformations and matrix multiplication. FIS 2.1–2.3.
(4) Coordinates of a vector space. Change of basis. Inverse of a linear transformation. Homogeneous linear differential equations with constant coefficients. FIS 2.4–2.5, 2.7.
(5) Elementary row/column operations. Rank. Matrix inverse. Reduced row echelon form. Gaussian elimination. FIS 3.1–3.4.
(6) Determinants. FIS 4.1–4.4.
(7) Eigenvalues. Eigenvectors and eigenspaces. Characteristic polynomial. Diagonalization. Cayley–Hamilton theorem. FIS 5.1–5.2, 5.4.
(8) Inner product spaces. Norms. Orthogonal basis. Gram–Schmidt orthogonalization process. Orthogonal complements. FIS 6.1–6.2.
(9) Adjoint. Normal and self-adjoint operators. Unitary operators. Orthogonal transformations. Spectral theorem. FIS 6.3–6.6.
(10) (Optional topic) Quadratic forms. Rotations and rigid motions. FIS 6.8, 6.11.
(11) (Optional topic) Matrix limits. Markov chains. FIS 5.3.
(12) (Optional topic) Google PageRank algorithm. Additional handout, not in book.
(13) (Optional topic) Heisenberg's uncertainty principle in quantum mechanics.

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