YALE UNIVERSITY DEPARTMENT OF MATHEMATICS Math 350 Introduction to Abstract Algebra Fall 2015

Problem Set # 2 (due at the beginning of class on Friday 25 September)

Reading: DF 2.1–2.5.

Problems:

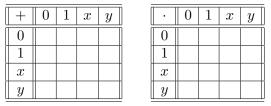
1. DF 2.1 Exercises 2, 6, 7, 8, 14, 15.

2. DF 2.2 Exercises 4, 7, 12, 14.

3. DF 2.3 Exercises 5, 9, 10, 12, 20, 21, 22, 23 (Hint: What does 22 tell you about the order of 5 in $(\mathbb{Z}/2^n\mathbb{Z})$?), 25, 26.

4. DF 2.4 Exercises 3, 6, 7, 8, 11 (Hint: What are the orders of elements in S_4 ?), 13, 14, 15, 16, 19.

5. Let $\mathbb{F}_4 = \{0, 1, x, y\}$. Prove that there are operations + and \cdot on \mathbb{F}_4 , such that 1 + x = y and $x^2 = y$, making \mathbb{F}_4 into a field. Note that the four elements of \mathbb{F}_4 are distinct! Essentially the problem is to fill out the addition and multiplication tables:



You already know certain rows and columns by properties of 0 and 1 in a field!