

YALE UNIVERSITY DEPARTMENT OF MATHEMATICS  
**Math 350 Introduction to Abstract Algebra**  
Fall 2015

Problem Set # 2 (due at the beginning of class on Friday 25 September)

**Reading:** DF 2.1–2.5.

**Problems:**

1. DF 2.1 Exercises 2, 6, 7, 8, 14, 15.
2. DF 2.2 Exercises 4, 7, 12, 14.
3. DF 2.3 Exercises 5, 9, 10, 12, 20, 21, 22, 23 (Hint: What does 22 tell you about the order of 5 in  $(\mathbb{Z}/2^n\mathbb{Z})$ ?), 25, 26.
4. DF 2.4 Exercises 3, 6, 7, 8, 11 (Hint: What are the orders of elements in  $S_4$ ?), 13, 14, 15, 16, 19.
5. Let  $\mathbb{F}_4 = \{0, 1, x, y\}$ . Prove that there are operations  $+$  and  $\cdot$  on  $\mathbb{F}_4$ , such that  $1 + x = y$  and  $x^2 = y$ , making  $\mathbb{F}_4$  into a field. Note that the four elements of  $\mathbb{F}_4$  are distinct! Essentially the problem is to fill out the addition and multiplication tables:

$+$	0	1	$x$	$y$
0				
1				
$x$				
$y$				

$\cdot$	0	1	$x$	$y$
0				
1				
$x$				
$y$				

You already know certain rows and columns by properties of 0 and 1 in a field!