

Problem Set # 3 (due at the beginning of class on Friday 2 October)

Notation: A subgroup $K \subset G$ is called **normal** if $gxg^{-1} \in K$ for every $x \in K$ and $g \in G$. It is common notation to denote a subgroup by $K \leq G$ and a normal subgroup by $K \trianglelefteq G$.

Reading: DF 2.5–3.2.

Problems: (Starred* problems are strongly recommended!)

1. DF 2.5 Exercises 4, 10, 13*, 14*.
2. DF 3.1 Exercises 5, 6, 7, 8, 9, 10, 11, 14*, 17*, 22, 25, 26* (you actually already did part a), 32, 34, 36*, 41*, 42.
3. DF 3.2 Exercises 4, 9, 14, 16, 19, 21*, 22* (Euler's theorem!).
4. *Tricks with Euler's theorem**. You can only use pencil and paper!
 - (a) Find the remainder after dividing 99^{999999} by 23.
 - (b) Prove that every element of $(\mathbb{Z}/72\mathbb{Z})^\times$ has order dividing 12. (Hint: This is better than what a straight application of Euler's theorem will give you! Try applying Euler's theorem to a pair of relatively prime divisors of 72.)
 - (c) Find the last two digits of the huge number $3^{3^{\dots^3}}$ where there are 2015 threes appearing! (Hint: Do five nested applications of Euler's theorem.)