Yale University Department of Mathematics
Math 350 Introduction to Abstract Algebra
Fall 2016

Problem Set # 8 (due at the beginning of class on Monday 14 November)

Reading: DF 5.4, 5.5.

Problems:

1. DF 5.4 Exercises 9, 13*.

2. DF 5.5 Exercises 5, 6*, 7*, 8, 10*, 11*, 13*, 20, 23.

3. Prove that if n is odd, then D_{4n} is isomorphic to $D_{2n} \times Z_2$.

4. *Subgroups of fields.* Let F be a field.

(a) Prove that any polynomial of degree n with coefficients in F has at most n roots in F. Hint. Induction on the degree of the polynomial.

(b) Prove that every finite subgroup of the multiplicative group $F^\times = F \smallsetminus \{0\}$ is cyclic. Hint. Fix a prime p dividing the order n of the subgroup, let q be the highest power of p dividing n. Consider the map $F^\times \to F^\times$ defined by raising to the n/q power. By considering the orders of the kernel and image of this map, conclude that there is an element of this subgroup of order q (at some point, you'll need the previous part). Do this for each prime dividing n and then find a generator for the group.

(c) Prove that if F is a finite field then $F^\times$ is cyclic. For each field F having at most 7 elements, find an explicit generator of $F^\times$.