Yale University Department of Mathematics
Math 350 Introduction to Abstract Algebra
Fall 2017

Problem Set # 3 (due at the beginning of class on Friday 29 September)

Notation: Given a subset A of a group G, the **subgroup generated by** A is the subset $\langle A \rangle$ in G of all products of powers of elements in A, which is actually a subgroup of G. The main result of DF 2.4 is that $\langle A \rangle$ coincides with the intersection of all subgroups of G that contain A, in other words, $\langle A \rangle$ is the "smallest" subgroup of G containing A.

Reading: DF 2.2–2.5.

Problems:

1. DF 2.2 Exercises 6, 7*, 12, 14.

2. DF 2.4 Exercises 6, 7, 8, 9*, 11* (Hint: What are the orders of elements in S_4?), 13, 12*, 14*, 15, 19.

3. DF 2.5 Exercises 4, 10, 12*, 14*, 15.

4. DF 3.2 (You do not need to read DF 3.2 in order to do these exercises, just Lagrange's Theorem.) Exercises 8*, 9, 13*, 16*, 22* (Euler's theorem!).

5. Show that for all $n, m \geq 1$, the group S_{n+m} contains a subgroup isomorphic to $S_n \times S_m$. Conclude that $n!m!$ divides $(n+m)!$.

6. Let H be the subgroup of S_4 generated by the 3-cycles. Show that there exists a positive integer n, with n dividing the order of H, and such that H has no subgroup of order n.

7. *Tricks with Euler's theorem.* You can only use pencil and paper!

(a) Find the remainder after dividing 99^{999999} by 29.

(b) Prove that every element of $(\mathbb{Z}/72\mathbb{Z})^\times$ has order dividing 12. (Hint: This is better than what a straight application of Euler's theorem will give you! Try applying Euler's theorem to a pair of relatively prime divisors of 72.)

(c) Find the last two digits of the huge number $3^{3^{3^{\cdot^{\cdot^{\cdot^{3}}}}}}$ where there are 2017 threes appearing! (Hint: Do nested applications of Euler's theorem.)

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