Yale University Department of Mathematics
Math 370 Fields and Galois Theory
Spring 2018

Problem Set # 1 (due in class on Thursday 25 January)

Reading: GT 2, 3.

Problems:

1. GT Exercises 2.5, 2.6.
These lead you through an algebraic proof of the fact that if a polynomial with coefficients in $\mathbb{C}$ gives the zero function, then it must be the zero polynomial (in the text, there is a proof using calculus, so don't assume this). As a consequence, we get that the map $\mathbb{C}[x] \to \mathrm{Map}(\mathbb{C}, \mathbb{C})$ is injective. While this can fail for polynomials over other fields, try to see what are the minimum set of requirements on the base field to make this algebraic argument work.

2. GT Exercises 3.1, 3.2, 3.3 (do only parts d and e for all of these).

3. GT Exercises 3.4 and also factor each of these polynoimals into irreducibles.

4. *Irreducible polynomials over finite fields*
Let $\mathbb{F}_3$ be the field with three elements.

(a) Determine all the monic irreducible polynomials of degree ≤ 3 in $\mathbb{F}_3[x]$.

(b) Determine the number of monic irreducible polynomials of degree 4 in $\mathbb{F}_3[x]$.

5. Prove that that two polynomials $f(x)$ and $g(x)$ in $\mathbb{Z}[x]$ are relatively prime in $\mathbb{Q}[x]$ if and only if the ideal $(f(x), g(x)) \subset \mathbb{Z}[x]$ contains a nonzero integer.

6. For this problem, recall the Math 350 problem on symmetric polynomials, and in particular, the parts on the generic polynomial and power sums.

(a) If x, y, z are complex numbers satisfying

$$x + y + z = 1, \qquad x^2 + y^2 + z^2 = 2, \qquad x^3 + y^3 + z^3 = 3,$$

then prove that $x^n + y^n + z^n$ is rational for any positive integer n.

(b) Calculate $x^4 + y^4 + z^4$.

(c) Prove that each of x, y, z are not rational. **Hint.** An earlier problem will not help. Try looking modulo all primes < 20.

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