Yale University Department of Mathematics
Math 373/573 Algebraic Number Theory
Spring 2019

Final Exam (due 5:30 pm on May 8)

Instructions: The L-functions and Modular Forms Database (LMFDB) is a fantastic resource for researchers in number theory. In particular, it contains a database of 19,707,196 number fields of degree $n \leq 23$:

http://www.lmfdb.org/NumberField/

You can quickly browse these entries by degree, discriminant, signature $[r, s]$, or class number. Alternatively, you can search for a specific number field in the database, either by entering the minimal polynomial of a generator, or by specifying various invariants. The entry for each number field contains all you might like to know, including: an integral basis (in terms of the fixed integral field generator given), the class group, unit group, Galois group of its normal closure, and even the splitting of primes up to 59. You can click on any piece of dotted underlined terminology to see the definition and/or notational input convention.

The point of this final exam is to explore the database of number fields and to provably verify the data contained in some of the entries by hand, using the theoretical tool we have. You are not allowed to work with other people, but you can use internet resources, books, and a basic calculator.

Problems:

1. What is your favorite number field $K/\mathbb{Q}$ in the database and why?

2. Choose one of the 684,216 real quadratic number fields $K/\mathbb{Q}$ in the database and verify, by hand, all the nontrivial invariants listed that we have defined in class: ring of integers, discriminant, signature, class group, and unit group including fundamental unit. Aside from aesthetics, you might want to choose the number field based on the theoretical tools we have at our disposal to compute these invariants. The number field you choose must have discriminant ≥ 1000 and class number ≥ 3. Make sure to provide the LMFDB label for your choice.

3. Choose one of the 867,744 cubic number fields $K/\mathbb{Q}$ in the database and verify, by hand, all the invariants listed that we have defined in class: ring of integers, discriminant, signature, Galois group of the normal closure, class group, and unit group including fundamental unit(s). It will be crucial to choose a number field that strikes a balance between the difficulty in employing the various theoretical tools we have at hand. The number field you choose must be ramified over at least two primes and must have nontrivial class group. Make sure to provide the LMFDB label for your choice.

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