

YALE UNIVERSITY DEPARTMENT OF MATHEMATICS
MATH 608 INTRODUCTION TO ARITHMETIC GEOMETRY
FALL SEMESTER 2018

Instructor: Asher Auel	Lecture: LOM 201
Office: LOM 210	Time: Mon Wed 1:00 – 2:15 pm
Web-site: http://users.math.yale.edu/~ael/courses/608f18/	

Introduction: This course will explore some of the major themes in arithmetic geometry. Topics will include Galois cohomology and descent, principal homogeneous spaces, quadratic forms and the Brauer group, Milnor K -theory, as well as the existence of rational points. Of particular interest will be varieties over finite fields, number fields, and function fields.

Grading: The final grade is based on class participation, problem sets, and a final exam.

Prerequisites: Prior experience with algebra and Galois theory will be necessary. Some exposure to algebraic geometry will be useful, but may be taken in conjunction with a beginning algebraic geometry course.

Topics covered: Subject to change.

- (1) Fields of interest (finite fields, p -adic fields, number fields, and function fields) and Galois theory.
- (2) Projective space and the basics of affine and projective algebraic varieties. Rational points.
- (3) Quaternion algebras and conics. Wedderburn's Theorem. Witt's Theorem. Quadratic reciprocity. Local-global principles for number fields and rational function fields.
- (4) Galois cohomology. Hilbert Theorem 90. Torsors and nonabelian H^1 . Linear algebraic groups.
- (5) Central simple algebras. Artin–Wedderburn theory. The Brauer group.
- (6) Milnor K -theory. Resultus of Merkurjev–Suslin. Milnor Conjecture and Block–Kato Conjecture.
- (7) Tsen–Lang theory. C_i -fields. C_1 -fields have trivial Brauer group. Chevalley's Theorem. Tsen's Theorem. Lang's Theorem. Applications to central simple algebras and quadratic forms.
- (8) Descent obstruction and Brauer–Manin obstruction.