Yale University Department of Mathematics
Math 608 Introduction to Arithmetic Geometry
Spring Semester 2016

Instructor:	Asher Auel	**Lecture:**	LOM 20
Office:	LOM 210	**Time:**	Tue Thu 10:30 – 11:35 pm
Web-site:	http://math.yale.edu/~auel/courses/608s16/		

Introduction: This course will explore some of the major themes in arithmetic geometry, i.e., the study of algebraic varieties over arbitrary rings and fields. Topics will include quadratic forms and the Brauer group, Galois cohomology and descent, principal homogeneous spaces and local-global principles, as well as the existence of rational points and rational parametrizations. Of particular interest will be varieties over finite fields, valued fields, real closed fields, Ci-fields, number fields, and functions fields. Recurring objects of study throughout the course will be quadric hypersurfaces and rational surfaces from an arithmetic and geometry point of view.

Grading: Your final grade will be class participation and occasional problem sets. There will be no exams.

Prerequisites: Courses in algebra and Galois theory are the only serious prerequisites. Some algebraic number theory would be helpful at points, but not strictly necessary. Can be taken in conjunction with a beginning algebraic geometry course.

Topics covered: Subject to change.

(1) Background material. Discrete valuation rings and completions. Hensel's Lemma. Ostrowski's Theorem for number fields and rational function fields. Integral closures. Finite fields, quadratic reciprocity, and Hilbert symbols. Projective space and plane curves.

(2) Quaternion algebras and conics. Wedderburn's Theorem. Witt's Theorem. Local-global principles for number fields and rational function fields.

(3) Central simple algebras. Brauer group.

(4) Quadratic forms. Classical invariants: discriminant and Clifford invariant. u-invariant of a field. Rational points on projective quadrics.

(5) Tsen-Lang theory. C_i-fields. C_1-fields have trivial Brauer group. Chevalley's Theorem. Tsen's Theorem. Lang's Theorem. Applications to the u-invariant.

(6) Torsors. Galois cohomology. Hilbert Theorem 90. Kummer theory. Nonabelian H^1. Linear algebraic groups.

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