

DARTMOUTH COLLEGE DEPARTMENT OF MATHEMATICS  
**Math 71 Algebra**  
Fall 2021

Midterm 2 Review Sheet

**Directions:** The second midterm exam will be handed out, in physical paper form, at the end of class on Tuesday, October 26th and will be picked up at the beginning of class on Thursday, October 28th. You must spend a maximum of 2 hours, in a single continuous stretch, working on the midterm exam, and you must write your starting time and ending time at the top of your exam paper. No electronic devices, notes, external resources, nor textbooks will be allowed to be used while you are working on the exam. On all problems, you will need to write your thoughts/proofs in a coherent way to get full credit.

**New topics covered (since the last midterm) and practice problems:**

- Group actions. Orbit-stabilizer theorem. Left multiplication action. Conjugation action. Conjugacy classes. Center. Class equation.  
DF 1.7 Exercises 1–3, 5–6, 8–13, 20, 21, 23; DF 4.1 Exercises 1–6; DF 4.2 Exercises 1–3. DF 4.3 Exercises 2–3, 7, 10–12.
- Subgroups. Cyclic subgroups. Centralizers. Generators. Lattice of subgroups.  
DF 2.1 Exercises 1–5, 14; DF 2.2 Exercises 1–2, 7; DF 2.3 Exercises 1–5, 10–14. DF 2.4 Exercises 6–9; DF 2.5 Exercises 4, 9–10, 15.
- Quotient groups. Cosets. Isomorphism theorems. Composition series.  
DF 3.1 Exercises 6–13, 33–35; DF 3.2 Exercises 8, 13–17, 21–23; DF 3.3 Exercises 1, 8; DF 3.4 Exercises 1–2 (just do  $D_8$ ).
- Alternating groups.  
DF 3.5 Exercises 1–4, 6, 8, 9, 12,

**Practice exam questions (not representative of the length of the exam):**

1. There will be several True/False problems covering a range of topics so far: isomorphic groups, orders of elements, homomorphisms, Lagrange's theorem, and group actions.
2. Make yourself familiar with the lattice of subgroups and conjugacy classes of some common groups, including  $\mathbb{Z}/n\mathbb{Z}$ ,  $S_3$ ,  $D_8$ ,  $A_4$ . Know all the normal subgroups and what the quotients are.
3. How many elements of order 6 are there in  $A_5$  and  $A_6$ ?
4. Show that  $2021^{2021} - 5$  is divisible by 24.
5. Let  $V_4 \subset S_4$  be the subset consisting of  $(2, 2)$ -cycles and the identity. Prove that  $V_4$  is a normal subgroup and that the quotient  $S_4/V_4$  is isomorphic to  $S_3$ .