Dartmouth College Department of Mathematics
Math 71 Algebra
Fall 2023

Problem Set # 3 (due via Canvas upload by 5 pm, Wednesday, October 18)

Reading: DF 2.2–2.5, 3.1–3.3.

Problems:

1. DF 2.4 Exercises 6, 7, 8, 9* (You already know how to compute the order of $\mathrm{SL}_2(\mathbb{F}_3)$, so do it!), 11* (Hint: What are the orders of elements in S_4?), 12*, 13, 14*, 15, 19.

2. DF 2.5 Exercises 4, 10, 12*, 14*, 15.

3. DF 3.1 Exercises 5–12, 14, 17*, 22, 34, 36*, 40, 41*, 42.

4. DF 3.2 Exercises 4*, 5, 8*, 13* (prove that no nonidentity element in this D_8 commutes with any nonidentity element of $\langle(123)\rangle$), 16, 22* (Euler's theorem!).

5. Show that for all $n, m \geq 1$, the group S_{n+m} contains a subgroup isomorphic to $S_n \times S_m$. Conclude that $n!m!$ divides $(n+m)!$.

6. *Tricks with Euler's theorem.* You can only use pencil and paper!

(a) Prove that every element of $(\mathbb{Z}/72\mathbb{Z})^\times$ has order dividing 12. (Hint: This is better than what a straight application of Euler's theorem will give you! Try applying Euler's theorem to a pair of relatively prime divisors of 72.)

(b) Find the last two digits of the huge number $3^{3^{3^{\cdot^{\cdot^{\cdot^{3}}}}}}$ where there are 2023 threes appearing! (Hint: Do nested applications of Euler's theorem.)

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