

**Math 71 Algebra**

Fall 2025

Quiz 1 Review Sheet

**Directions:** The first in-class quiz will be Tuesday, October 7, during the last 15 minutes of class. No electronic devices, notes, external resources, nor textbooks will be allowed during the quiz. The quiz will not feature problems that require full proofs to be written. However, on all problems, you will need to briefly explain your thoughts in a coherent way, and which theorems you are appealing to, to get full credit.

**Topics covered and practice problems:**

- Basic set theory and functions (injections, surjections, bijections). Definition of a group. Presentation with generators and relations. Modular arithmetic. Multiplicative group modulo  $n$ .  
DF 0.1 Exercises 4–6; DF 0.2 Exercises 7, 8, 11; DF 0.3 Exercises 4, 5, 7, 9; DF 1.1 Exercises 1, 2, 5–9, 12–14, 21, 28, 31, 36.
- Dihedral groups. Symmetric groups. Disjoint cycle decomposition of permutations. Matrix groups over a field.  
DF 1.2 Exercises 4–6, 9; DF 1.3 Exercises 9–19; DF 1.4 Exercises 7, 10.
- Homomorphisms and isomorphisms. Kernel. Image.  
DF 1.6 Exercises 3–9, 11, 15–16, 19, 21–22, 25.
- Subgroups. Cyclic subgroups.  
DF 2.1 Exercises 1–5, 14; DF 2.3 Exercises 1–5, 10–14.

**More practice problems (not representative of the length of the quiz):**

1. There will be True/False problems covering a range of topics so far: properties of groups, orders of elements, homomorphisms, and Lagrange's theorem.
  2. Let  $V_1 \subset \mathbb{R}^2$  and be the subset of all points whose slope is an integer, together with the integer valued points on the vertical axis. Let  $V_2 \subset \mathbb{R}^2$  be the subset of all points whose slope is a rational number, together with the rational number valued points on the vertical axis. Determine if  $V_1$  and/or  $V_2$  is a subgroup of  $\mathbb{R}^2$ , with usual addition.
  3. Write down a nontrivial homomorphism  $\mathbb{Z}/36\mathbb{Z} \rightarrow \mathbb{Z}/48\mathbb{Z}$  and compute its image and kernel. Can you find an injective homomorphism or a surjective homomorphism?
  4. How many elements of order 6 are there in  $\mathbb{Z}/5\mathbb{Z}$ ,  $S_6$ , and  $(\mathbb{Z}/7\mathbb{Z})^\times$ ?
  5. Prove that  $11^{104} + 1$  is divisible by 17.
  6. Show that the set of nonzero matrices of the form
 
$$\begin{pmatrix} a & 3b \\ b & a \end{pmatrix}$$
 is a cyclic subgroup of  $\text{GL}_2(\mathbb{F}_5)$ . What is the order of this subgroup?
  7. Find a dihedral group with a subgroup isomorphic to  $(\mathbb{Z}/7\mathbb{Z})^\times$ .
- Bonus.** Classify all such dihedral groups.