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Prove that there is a unique group of order 4

To prove that there is a unique group of order 4, we need to show that any two groups of order 4 are isomorphic and therefore identical.

First, let's consider a group of order 4, denoted as G. There are two possibilities for the structure of a group of order 4:

1. G is isomorphic to the cyclic group of order 4, which is denoted as C4, and has the elements {e, a, a^2, a^3}, where "e" is the identity element, and "a" is a generator of the group.
2. G is isomorphic to the Klein four-group, denoted as V or K4, and has the elements {e, a, b, c}, where "e" is the identity element, and "a," "b," and "c" are non-identity elements such that a^2 = b^2 = c^2 = e, and ab = ba = e, ac = ca = e, and bc = cb = e.

Now, let's prove that any group of order 4 is isomorphic to either C4 or K4:

Case 1: G is cyclic of order 4 (C4):
In this case, G is generated by a single element "a," and G is isomorphic to C4. The group C4 is unique up to isomorphism.

Case 2: G is not cyclic, so it must be the Klein four-group (K4):
If G is not cyclic, it must have elements that do not generate the entire group. Since G has order 4, it can only be isomorphic to K4. The Klein four-group is unique up to isomorphism.

Therefore, any group of order 4 is either isomorphic to C4 or K4, and both C4 and K4 are unique up to isomorphism. This proves that there is a unique group of order 4 up to isomorphism, as any other group of order 4 must be isomorphic to one of these two groups.

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Is there a unique group of order 4?

Yes, there is a unique group of order 4, up to isomorphism. This group is called the cyclic group of order 4 and is denoted as C4. The elements of C4 can be represented as {e, a, a^2, a^3}, where "e" is the identity element, and "a" is a generator of the group.

C4 is unique because any group of order 4 is isomorphic to C4. This means that while there may be different ways to represent the group elements or write its operation table, any group with four elements and the same multiplication structure is isomorphic to C4. Therefore, there is a unique group of order 4 up to isomorphism.

Regenerate

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