PROBABILITY

Rubik's Cube

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MIDTERM 1 #3

POINTS TO TALK ABOUT

Reviewing the question
Looking at my response
Analyze personal error
Rework question correctly

THE QUESTION

A Rubik's cube is a cube with its exterior painted, and can be disassembled into 27 small equal cubes.

Your friend put small cubes in a bag, ask you to close your eyes, draw one small cube at random, and place it on a table.

(a) What is the probability that you cannot find any painted face after you open the eyes? (Hint: You can only see 55 faces of a small cube, the face down side on the table is not visible.)
(b) After you open your eyes and you indeed find none of 55 faces painted, what is the probability that the face down side is painted?

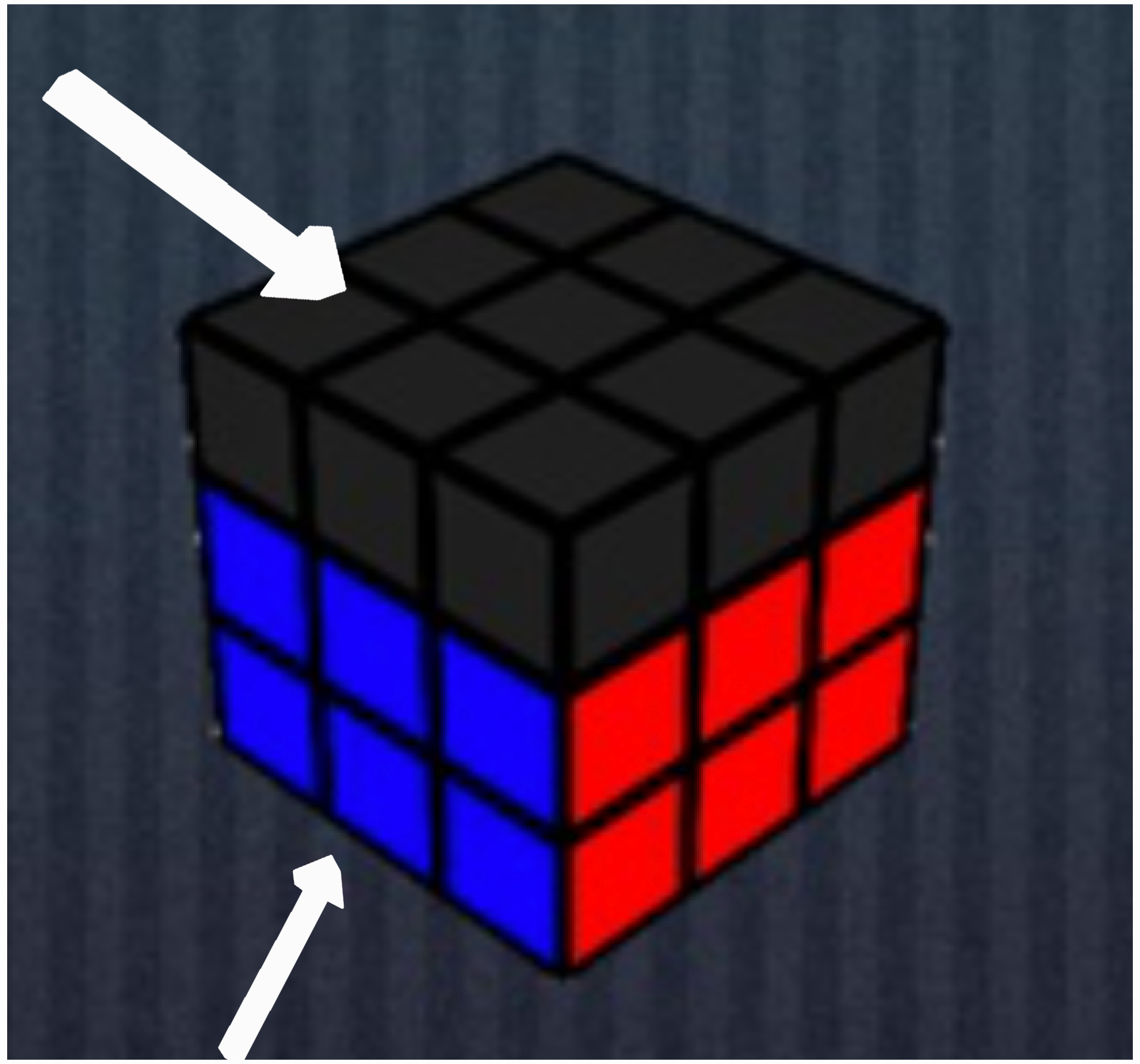
You need to use the set theory to describe all the events you plan to compute. Show all your work. Answers without reasoning may result in zero partial credit.

PROPERTIES

- Sample space: {small cubes}
- $n(\Omega)$= 27 small cubes
- Since only the exterior is painted, we can group cubes according to the number of painted faces they have

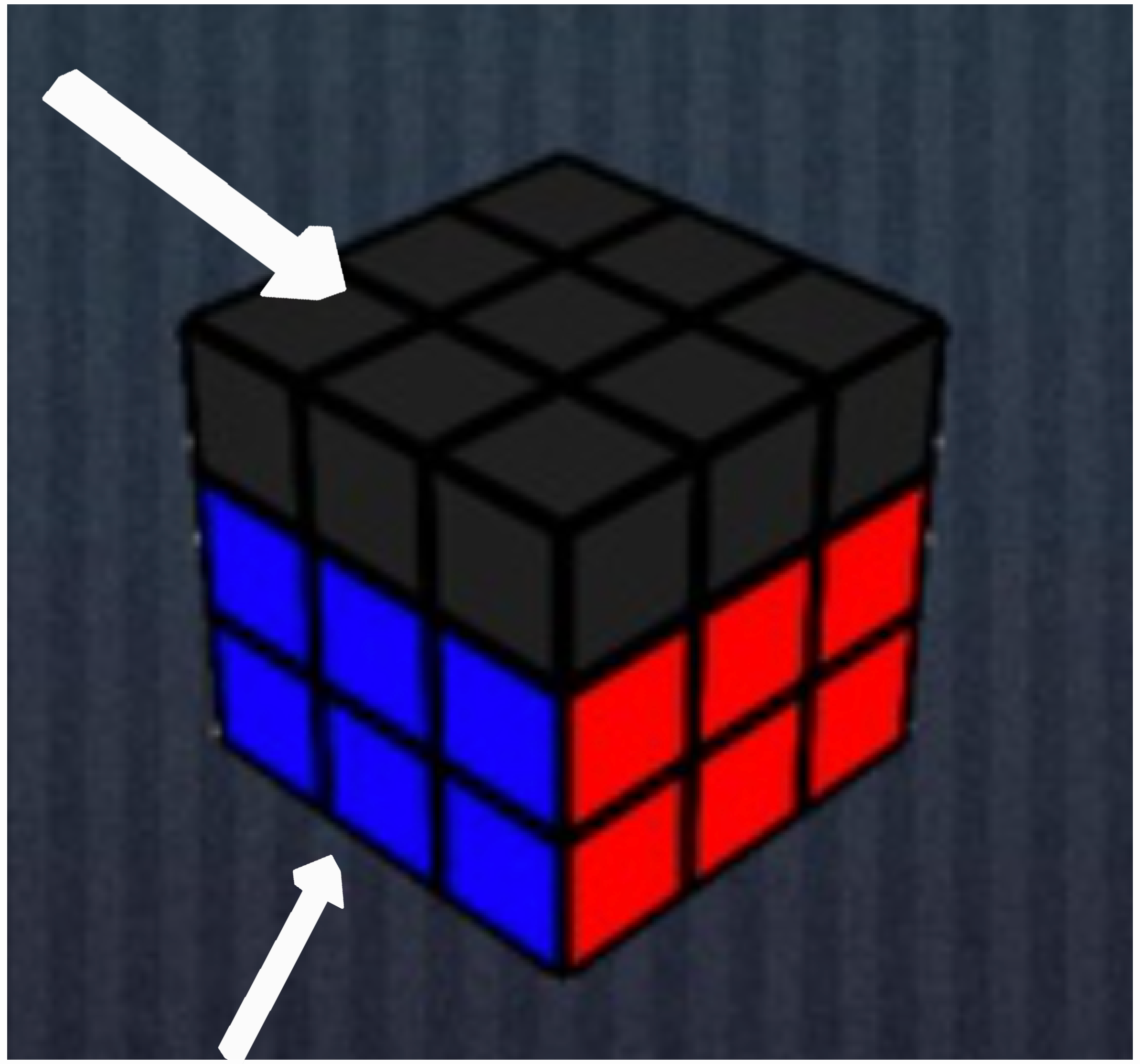
MY APPROACH

- Dividing the cube into 3 sets of 9 cubes
- The two outer sets of the Rubik's cubes would be identical so I grouped them together to get the number of painted faces.



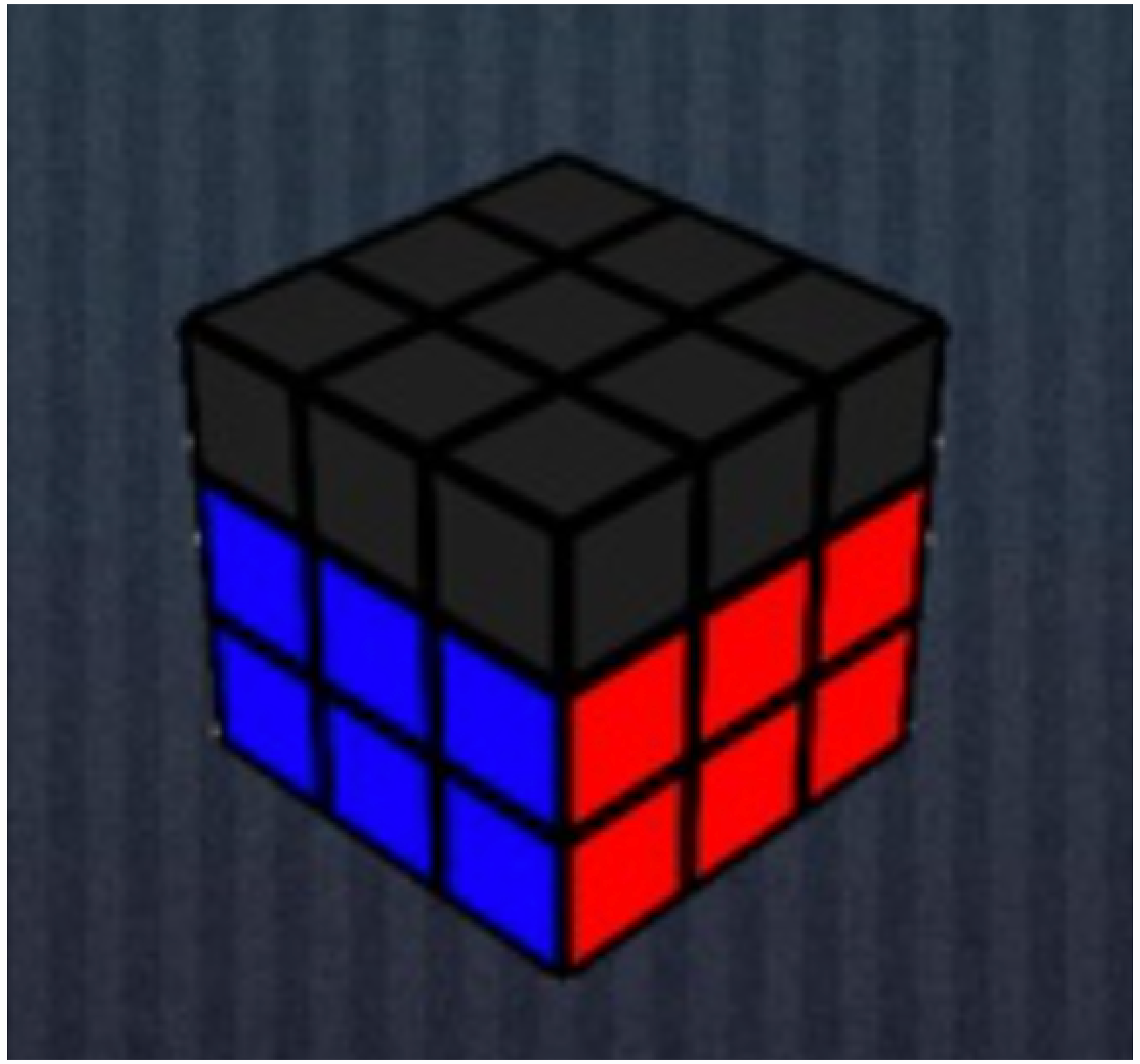
MY APPROACH

- Each of the 2 outer sets have 9 cubes: 4 corner pieces have 3 painted faces, 4 end pieces have 2 painted faces, 1 middle piece has 1 painted face.
- For the middle set: the four corner pieces have 2 painted faces, 4 end pieces have 1 painted face and the middle piece has no painted faces.

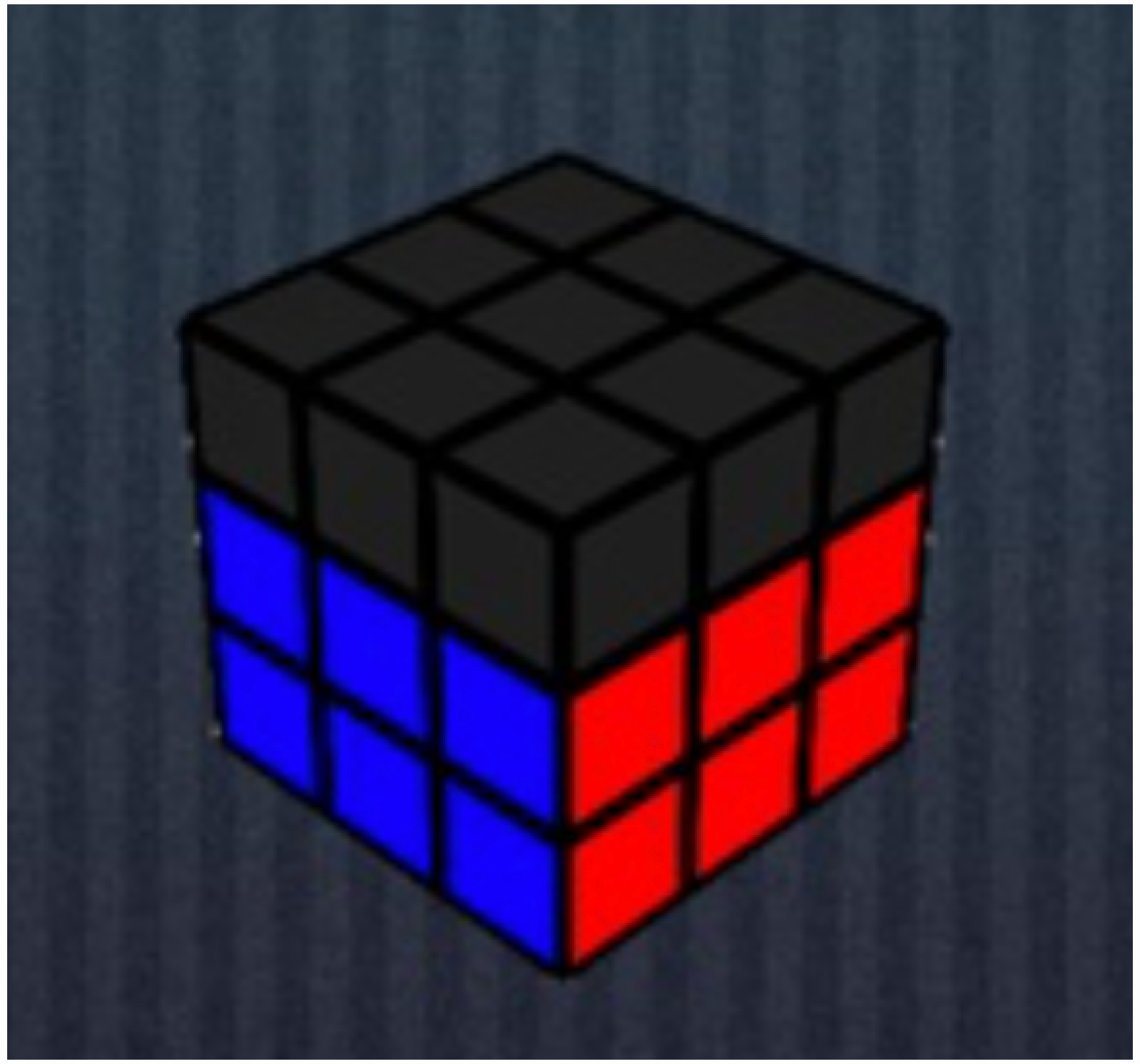


PART A

- I chose to divide them up by location in a row, rather than the number of painted faces they had
- Let A= {corner pieces}
- n(A) = 12
- Let B= {end pieces}
- n(B) = 12
- Let C= {middle pieces}
- n(C) = 3
- C1 ={exterior middle pieces}
- C2= {interior middle pieces}



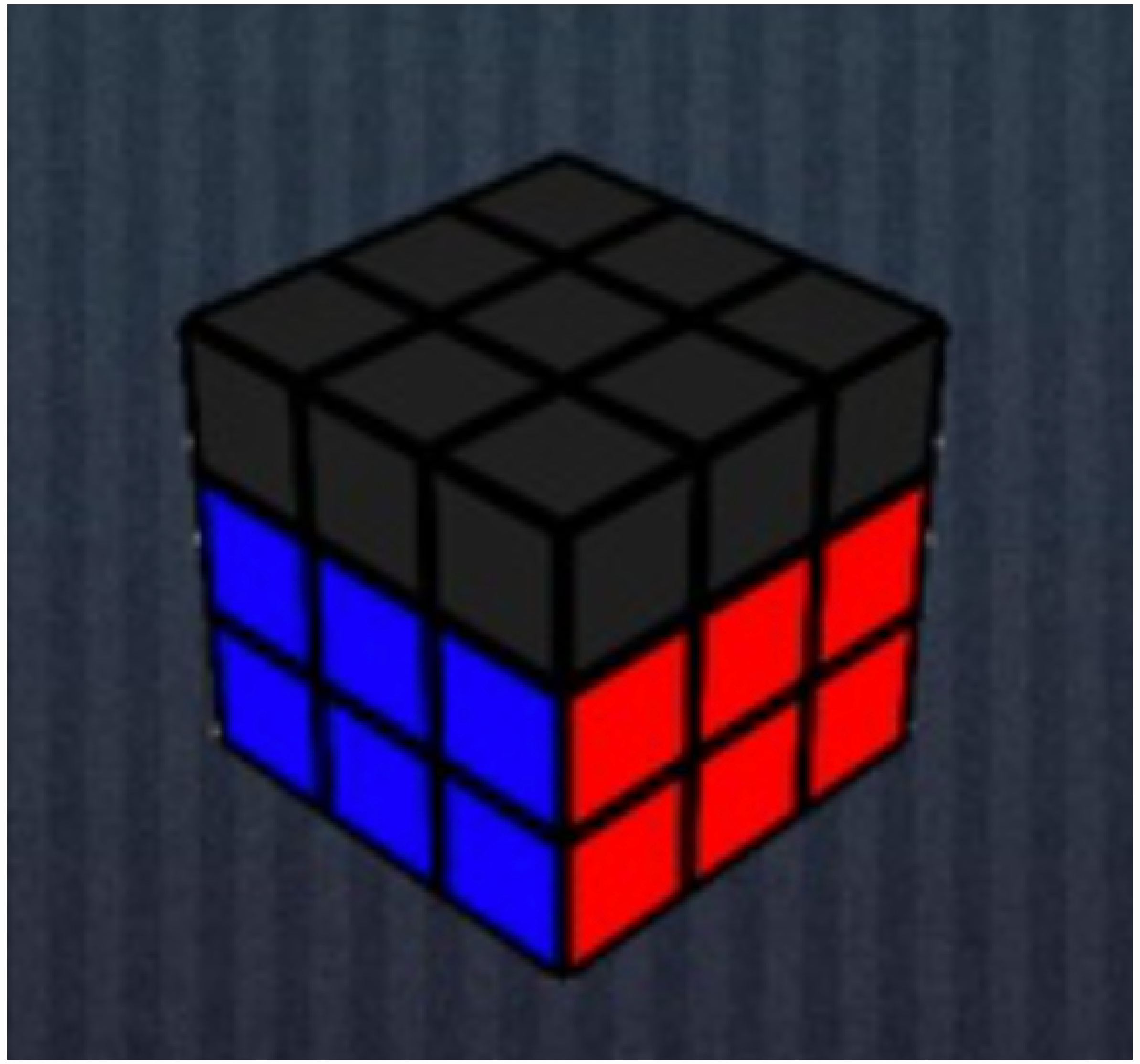
PART A



- After opening eyes, if there was no painted face, I assumed it must have been one of the middle pieces that had only one painted side or the interior middle piece that had no painted faces.
- $P(C) = n(C)/n(\Omega) = 3/27 = 1/9$

CORRECTED PART A

- 8 cubes have 3 faces painted, 12 cubes have 2 faces painted, 6 cubes have 1 face painted and 1 cube has no face painted.
- Let Ci represent the event that we choose a cube with i faces painted
- Let E= {no painted faces are visible}
- We want to find the probability that we choose a cube with 1 or 0 faces painted and that no painted face is visible



- Using total probability:
- $P(E) = P(E \cap C1) + P(E \cap C0)$

$$= (6/27)(1/6) + 1/27$$

$$= 2/27$$

ALTERNATE METHOD

- Make a subset of the sample space with A= {cubes with 1 face painted}, B= {cubes with no face painted}
- n(A) = 6
- n(B) = 1
- Let E= {no painted faces}

- Using Combinations and Multiplication Principle, P(E) can be calculated.

- P(E)= $$\frac{\left(\binom{6}{1} * \frac{1}{6}\right) + \left(\binom{1}{1} * 1\right)}{\binom{27}{1}}$$

=(1+1)/27

=2/27

PART B

- Let E= {no painted faces visible}
- Let F= {face down is painted}
- We are trying to find the probability that the face down is painted, given that no painted faces were visible
    - $P(F|E) = P(F \cap E) \,/\, P(E)$

      $= P(C1 \cap E) \,/\, P(E)$

      $= (6/27)(1/6) \,/\, (2/27)$

      $= 1/2$