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Carl Pomerance
John G. Kemeny Parents Professor of Mathematics, Emeritus

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Prof. James Talmage Reid
Chair of the Department of Mathematics
The University of Mississippi
P.O. Box 1848
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Dear Prof. Reid:

At your request I am commenting on the research credentials of Micah Milinovich who is being considered for promotion to Professor. I first met Dr. Milinovich at a small conference at the University of Maine in 2007, when he was still a grad student. I recall being impressed with his mathematical maturity at that early point in his career. Since then we have met at other conferences, the last time being a year ago in Memphis.

Dr. Milinovich is one of the top researchers in the US in analytic number theory. This field was essentially begun by Riemann in the mid nineteenth century. He discovered that the distribution of the prime numbers among all positive integers is intimately connected with what we now call the Riemann zeta-function. It is the sum of $1/n^s$ over all positive integers n. Here s is a complex variable. Denoted $\zeta(s)$, the infinite sum converges when the real part of s is larger than 1 to an analytic function that may be continued to the whole complex plain, but for a simple pole at $s = 1$. The connection to prime numbers is perhaps not so mysterious since an alternate formulation of $\zeta(s)$ is the product of $1/(1-p^{-s})$ over all primes p.

The famous Riemann Hypothesis, one of the million-dollar prizes offered by the Clay Mathematical Institute for a proof, asserts that all of the zeros of $\zeta(s)$ when the real part of s is positive, have real part exactly $1/2$. This assertion is essentially equivalent to the conjecture of Gauss that $\int_2^x dt/\ln t$ is an excellent approximation to the number of primes $p \le x$. Thus, we see the number theory in the primes and the analysis in $\zeta(s)$.

Riemann's approach has been generalized to other problems of arithmetic interest, such as elliptic curves and algebraic number fields, the latter being connected with some of of Dr. Milinovich's work. These are subfields of the complex numbers that have finite dimension when considered as vector spaces over the field of rational numbers. (A famous example is the field of Gaussian rationals which consists of all numbers $a + bi$ where a, b are rational numbers and $i^2 = -1$.) One can do arithmetic in algebraic number fields, but each field has its own twists. In particular playing the role of primes are now prime ideals. And ordinary prime numbers "split" into these prime ideals. A nice case is when the prime splits completely. An example is the prime 5, which splits as $1 \pm 2i$ in the Gaussian rationals, where the prime 3 does not split completely there. Given a field, one can ask for the least prime that splits completely. Earlier work had gotten non-trivial upper bounds when the field was "nice" in that the group of symmetries of the field was abelian. The much harder non-abelian case was recently dealt with in a very impressive joint paper of Dr. Milinovich with his former grad student Zhenchao Ge and Paul Pollack.

I am also very impressed with the recent paper of Dr. Milinovich with Carneiro and Soundararajan on prime gaps. Here they have made progress on something that has resisted any improvements in a century. If p_n denotes the n-th prime number, the question is how large can $p_{n+1} - p_n$ grow as a function of n. Of course it is sometimes quite small, and usually it is of magnitude $\ln n$, but occasionally it's larger. The Riemann Hypothesis implies that it is at most a constant times $\sqrt{p_n}\ln n$. Here, the three of them improved the value of the constant in this upper bound, lowering it by a bit more than 16%. This might seem trivial, but as I've said it is the only progress on this famous problem in a century. Perhaps this will release a floodgate of new ideas.

Dr. Milinovich works very well in a collaborative setting, and he has been very successful with his students. I have heard his former student Caroline Turnage-Butterbaugh give some inspired talks and at least a small part of the credit should go to her former adviser!

I might like to compare Dr. Milinovich with Paul Pollack at the University of Georgia. They both received their Ph.D.'s in 2008, they both work in analytic number theory, and they are both rising stars. Where Pollack is more eclectic, Milinovich delves deeper. Pollack was just promoted to full professor at UGA, and I think an equivalent case can be made for Dr. Milinovich. (Full disclosure: Pollack was my advisee at Dartmouth.) I recommend him to you in my strongest terms. If I can be of further help, please don't hesitate to ask.

Sincerely yours,

Carl Pomerance

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