Root Problems in Ergodic Theory

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(Tea 3:30 pm Math Lounge)

Abstract

T is a measure-preserving transformation on finite measure space, (X, β, m), if $m(T^{-1}A) = m(A)$ for all measurable sets A in σ-algebra β. Do all such transformations T have a square root? (S is a square root of T if $S^2 = T$). More than 30 years ago this question was first answered, negatively, by Ornstein in 1967. Recently, Jonathan King provided a new positive answer to this question, by showing the generic transformation has square roots. King's work also poses new interesting questions. For example, is there a transformation T with square roots of all orders but no infinite chain of square roots? The answer to this question and others can be obtained by constructing measure-preserving actions of a certain class of groups, the Root Groups. Through discussing these problems I hope to illustrate some of the basic ideas of Ergodic Theory and show how the Root Groups, which are very useful as examples in undergraduate algebra courses, arose naturally in this context.

This talk should be accessible to graduate students.