

The Legacy of the Continuum Hypothesis

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102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

This talk will endeavour to tell an accessible and interesting story about the mathematical legacy of the continuum hypotheses. Formulated in the push toward rigorous foundations in the late 1800s, the continuum hypothesis is the statement that the size of the reals is the size of the smallest uncountable set. Regarded in its day as a pressing problem for all of mathematics, this conjecture resisted many attempts at resolution; even Godel was only able to establish a partial result. (He showed that it was consistent with the normal axioms of set theory.) A complete solution had to wait until the 1960s, when Paul Cohen invented a new technique of set-theoretic proof, the method of forcing, which he used to show that it was also consistent that the continuum hypothesis be false. Although ramified and cumbersome in its initial presentation, forcing was quickly distilled to an elegant, powerful technique, capable of addressing a wide variety of interesting problems. Today it is used to address topics in topology, computability theory, and analysis, as well as in set theory proper.

We will begin with a brief historical discussion of how the continuum hypothesis came to be formulated. This will be followed by an intuitive description of how the method of forcing works, and an heuristic account of how it can be used to address the continuum hypothesis as well as selected modern questions from analysis and set theory.

Throughout, the talk will focus on intuition, motivation, and context. It will be accessible to graduate students.