KdV, mKdV and eigenfunctions of Schroedinger operators

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Abstract

The Korteweg – de Vries equation (KdV) appears in hydrodynamics, electronics, plasma physics, fiber optics, biology, and other areas of mathematics and applications. It is a universal equation modeling propagation of nonlinear waves with dispersion but without dissipation of energy. It explains the appearance and behavior of solitons (solitary waves) which were first discovered and experimentally investigated by Scott Russell in 1834-1844, and later explained by Korteweg and de Vries in 1895.

Peter Lax discovered in 1968 that KdV can be rewritten in terms of two 1-dimensional differential operators L and A, where L is the Schroedinger operator, which has order 2, and A, which has order 3. (Later the pair of such operators L, A was named the Lax pair for KdV.) Lax pairs proved to be essential for all further developments, in particular, in the discovery of the inverse scattering method for the solution of KdV in classes of functions decaying at infinity with respect to the space variable.

It occurs that there is another representation of KdV, which is similar to the Lax one, but replaces the third order operator A by a family of first-order operators. One of the applications of this construction is the existence and uniqueness of solutions of the initial value problem for the modified KdV (mKdV) in classes of functions which can even grow at infinity with respect to the space variable.

This talk should be accessible to graduate students.