The Erdos–Renyi phase transition

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(Tea 3:30 pm Bradley Lounge)

Abstract

Some forty five years ago Paul Erdős and Alfred Rényi wrote "On the Evolution of Random Graphs." Erdős and Rényi recognized that the random graph $G(n,p)$ (n vertices, adjacency probability p) undergoes a fundamental change when $p \sim \frac{1}{n}$. Parametrizing $p = \frac{c}{n}$, while $c < 1$ all components are small and simple but when $c > 1$ a complex giant component has emerged. Today we recognize this as a phase transition. Phase transitions (= sudden change, e.g., freezing) appear in mathematical physics (e.g., bond percolation on Z^d), computer science (e.g., random k-SAT), branching processes, and elsewhere. We give a general discussion of them. For Erdős-Rényi percolation we can expand the $c = 1$ value and we explain why the "proper" parametrization" for the "critical window" is $p = n^{-1} + \lambda n^{-4/3}$.

We explore this percolation phenomenon from a variety of viewpoints. One new approach (joint with Remco van der Hofstad) involves a novel analysis of the Breadth First Search algorithm on the random graph $G(n,p)$.

This talk should be accessible to graduate students.